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EVIDENCE FROM THE BUSINESS ANGEL MARKET

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Entrepreneurs Meet Financiers*

Evidence from the Business Angel Market

Angela Cipollone[†] Paolo E. Giordani[‡]

Abstract

This paper estimates the process of *search and matching* between entrepreneurs and financiers in the business angel (BA) market. We hand-collect a new dataset from the BA markets of 17 developed countries for the period 1996-2014, and we estimate the aggregate matching function expressing the number of successful deals as a function of the number of potential entrepreneurs and of business angels. Empirical findings confirm the technological features assumed in the theoretical literature: positive and decreasing marginal returns to both inputs (*stepping on toes* effect), technological complementarity across the two inputs (*thick market* effect) and constant returns to scale. We discuss the theoretical and policy implications of these findings.

Keywords: Entrepreneurial finance, angel investors, matching function estimation, constant returns to scale.

JEL Classification: 031, C78, L26.

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1 Introduction

In recent years the market for entrepreneurial finance -that is, finance provided to risky, innovative ventures- has received increasing attention in the economic and financial literature (Chemmanur and Fulghieri, 2014). Although still representing a small fraction of the overall investments in innovation, funds provided by such institutions as venture capitalists and business angels are rapidly growing in size and in prominence.¹ For instance, a recent study by the OECD (2011) shows that the number of angel networks operating in the US and in Europe has roughly tripled in the ten years from 1999 to 2009. Moreover, the amount of investments provided by business angels and the number of deals have been increasing during the 2000s despite the financial crisis. Finally, these investments tend to be concentrated in the most innovative sectors. For instance, in Europe and in the US, biotechnology, ICT and environmental technologies absorb around 60% of the overall angel investments (OECD, 2011).

The market for entrepreneurial finance can be essentially described as one in which potential entrepreneurs search for funds to finance their ideas, and financiers (or capitalists) search for good ideas to finance. In this perspective, a start-up enterprise is the result of a successful matching between the demand and the supply side of this market.

Depicting entrepreneurial finance as a *search and matching* process between entrepreneurs and financiers, rather than via the traditional demand and supply apparatus, has become a consolidated practice in the economic literature of entrepreneurship, and it is rooted into solid theoretical foundations.² In the real world, market participants -whether entrepreneurs or financiers- are *heterogeneous* in their skills, location, beliefs, preferences etc., and they are not perfectly informed about all market characteristics.

¹Business angels (also called angel investors) refer to wealthy individuals that invest *their own funds* in entrepreneurial ventures, differently from venture capitalists which, instead, gather funds from institutional investors, such as pension funds. Because of that, the amount invested in each project by a business angel is, on average, considerably lower than the one invested by a venture capitalist. They, however, share the following crucial feature: they are expected to contribute to the project not only with financial investments but also with managerial and technical expertise (Gompers and Lerner, 1999). Recent key references in the (still small but growing) literature on business angels include Kerr et Al. (2014) and Lerner et Al. (2015).

²The inspiring idea of this stream of literature is clearly expressed in Phelps (2009). The most relevant contributions for our purposes include Inderst and Muller (2004), Michelacci and Suarez (2004), Wasmer and Weil (2004), Sorensen (2007), Cipollone and Giordani (2016), Silveira and Wright (2016).

As a result, the meeting of demand and supply of financial funds for entrepreneurship is a costly and time consuming process. Search theory is a modeling tool that allows one to catch the most salient features of such frictional and decentralized markets as the market for entrepreneurial finance.³

The complex process of search and matching between demand and supply is usually and conveniently represented at aggregate level via the use of a *matching function* (Petrongolo and Pissarides, 2001). The rationale behind the matching function is the possibility of capturing succinctly all market imperfections and all dimensions of heterogeneity without the need to specify them in detail.⁴ Indeed, whether the matching function is a valid representation of the market for entrepreneurial finance ultimately rests on its empirical adequacy. In the words of Petrongolo and Pissarides (2001, p. 392), "like the other aggregate functions [the] usefulness [of the matching function] depends on its empirical viability and on how successful it is in capturing the key implications of the heterogeneities and frictions in macro models". The main purpose of this paper is to estimate the aggregate matching function, representing the number of successful deals as a function of the number of would-be entrepreneurs and of financiers, in the *business angel* (BA) market, and to verify the technological features commonly assumed in the theoretical literature.

Most of the theoretical literature looking at entrepreneurial finance as a search and matching process assumes a well-behaved matching function (often in a Cobb-Douglas form) with the following technological characteristics: (i) positive and decreasing marginal returns to both inputs (entrepreneurs and financiers); (ii) a positive complementarity across the two inputs; (iii) constant returns to scale (CRS).⁵ The first and the second features are intuitive: the marginal effect of an increase in the number of entrepreneurs on the number of successful matches (i) is positive but decreasing in the number of entrepreneurs -and, of course, the same holds for financiers (*stepping on toes* effect), (ii) is increasing in the number of financiers, and *viceversa* (*thick market* effect). The third feature, instead, deserves special attention.

In theoretical models where the market entry decision is endogenous, the efficiency

³Prior to finance, search theory has been extensively used in several fields of economics, such as labor economics, monetary theory, and the theory of marriage. Rogerson et Al. (2005) contains a survey of applications for the labor market but also a list of references for applications in other fields.

⁴Few papers delve into the microeconomic foundations of the search and bargaining frictions in the entrepreneurial market. See, for instance, Silveira and Wright (2010, 2016), Chiu et Al. (2011).

⁵See, among others, Inderst and Muller (2004), Michelacci and Suarez (2004), Chiu et Al. (2011).

of the search equilibrium is closely related to the returns to scale of the matching function. In particular, if the matching function does *not* exhibit constant returns to scale, the resulting equilibrium is certainly inefficient.⁶ The assumption on the returns to scale is then critical to evaluate the role of the policy maker, given that equilibrium inefficiency is a classical argument in favor of policy intervention. Furthermore, theoretical literature has shown that the scale elasticity of the matching function "disciplines" the number of equilibria admitted by the model.⁷ In particular, if returns to scale are increasing, the model may admit more than one equilibrium, which would also point to an active role of the policy maker. Providing empirical evidence on the returns to scale is then useful to inform the debate on entrepreneurship policy (Parker, 2009). We go back to this issue in the concluding section.

In the theoretical section, we introduce formally the matching function and describe the technological characteristics commonly assumed. In Appendix A we embed this matching technology into a parsimonious model of search and matching between entrepreneurs and financiers. In particular, the model describes the financial market for innovation as a "fair" in which the two sides of the market can meet bilaterally and transform a "rough" entrepreneurial idea into a real start-up firm. While the model is a relatively straightforward extension of the classical Diamond's (1982) *cocunut* model to a *two-sided* search structure (which is why we relegate it to the appendix), it is however useful (i) to gain some perspective on the implications of the technological hypotheses usually made and (ii) to state rigorously the relationship between the number of equilibria and the returns to scale of the matching technology (Proposition 1), as well as to give an economic interpretation to equilibrium multiplicity.

We then estimate the matching process between entrepreneurs and financiers using a unique, hand-collected dataset on innovative projects financed by business angels. We collect yearly data for the period 1996-2014 across 17 developed countries on (i) the number of business angels (financiers), (ii) the number of projects submitted to them (potential entrepreneurs), (iii) the number of deals (successful matches). With these data we estimate the aggregate matching function for the business angel market, which takes the number of projects and angels as inputs and the number of deals as

⁶More precisely, homogeneity of degree one of the matching function is a necessary but not sufficient condition for the (constrained) Pareto efficiency of the search equilibrium. The so called *Hosios conditions* (stating that the input shares in the surplus of a match be equal to the input elasticities) must also be satisfied (see Hosios, 1990 for details).

⁷Classical references are Diamond (1982, 1984).

output.

We consider several specifications of the matching function. We start from a non-linear estimation of a log-CES-type matching function. We then estimate a log-Cobb-Douglas matching function. Finally, and in line with the empirical literature on matching function estimation, we consider the more general form of transcendental logarithmic (or simply "translog") matching function. Whatever the specific functional form assumed, the estimated function systematically holds the technological characteristics assumed in the theoretical literature: positive and decreasing marginal returns to both inputs and positive technological complementarity across the two inputs. We also test the returns to scale of the estimated matching function, and thus indirectly verify the empirical plausibility of multiple equilibria. Our findings point to a unitary scale elasticity of the matching function and thus offer some, admittedly preliminary, empirical evidence against equilibrium multiplicity in the business angel market.

The paper is organized as follows. In Section 2 we introduce formally the matching function and state our testable hypotheses. In Section 3 we carry out the empirical analysis. Section 4 concludes the paper.

2 Entrepreneurial Finance as a Search and Matching Process

The production process of new entrepreneurial ventures can be described via the following aggregate matching function:

$$M = M(L_E, L_K), \quad (1)$$

where M, L_E, L_K denote, respectively, the number of successful matches, would-be entrepreneurs and financiers.

Theoretical models of search and matching typically assume positive and decreasing marginal returns to both inputs, that is to say

$$(i) \partial M / \partial L_j > 0 \text{ and } (ii) \partial^2 M / \partial L_j^2 < 0 \text{ for } j = E, K, \quad (\text{Hp } 1)$$

implying that the number of matches is increasing in both inputs at decreasing rate. The rationale behind part (ii) of (Hp 1) is that of capturing a sort of stepping on toes effect in the entrepreneurial finance market, which implies that the individual

probability of being funded for an entrepreneur (M/L_E) is decreasing in the number of competing entrepreneurs (and the same for financiers).

A second commonly assumed technological feature deserving empirical scrutiny is the existence of a positive complementarity across the two inputs. Two inputs are technological complements when the marginal productivity of one input is increasing in the use of the other input, that is, when the cross-partial derivative of the matching function is strictly positive:

$$\partial M / \partial L_j \partial L_{-j} > 0 \text{ for } j = E, K. \quad (\text{Hp 2})$$

In our context, this would imply that the impact of one additional potential entrepreneur on the output (of funded business ventures) is increasing in the number of financiers, and viceversa. This assumption captures the existence of a thick market effect, whereby the individual probability of being funded for an entrepreneur (M/L_E) is increasing in the number of financiers (and *viceversa*).

Finally, the third common assumption made in the literature is the homogeneity of degree one of the matching function, that is:

$$M(aL_E, aL_K) = aM(L_E, L_K) \text{ for } a \in R_+. \quad (\text{Hp 3})$$

In the next section, we will verify (Hp 1), (Hp 2), and (Hp 3) against the available data from the business angel market. In Appendix A at the end of the paper, we further discuss these hypotheses and embed them into a formal model of search and matching. In particular, we formally show that, far from being innocuous, the third hypothesis is responsible for equilibrium uniqueness.

3 Estimating the Search and Matching Process

3.1 The Data on the Business Angel Market

A key challenge for the estimation of function (1) is the search of suitable data for our three variables of interest, L_E, L_K, M . As argued in the Introduction, the focus on the business angels market is justified by its growing importance in the financing of innovative entrepreneurs. On the other hand, the scarcity of empirical work is due to the lack of available data on the business angels' activity.⁸ This paper represents

⁸One recent exception is Lerner et Al. (2015) who gather data on 13 angel investment groups based in several countries to verify the effectiveness of angel investments in improving the outcomes and the performance of start-up firms in which they invest.

a first attempt to hand-collect the data necessary to explore the characteristics of the matching process in the business angel market.

In the US, the Center for Venture Research (CVR) at the University of New Hampshire reports yearly and quarterly information about the angel investor market in the US as a whole, providing details on the number of active investors, the overall investment size by industry and by stage of investments, the contribution to job creation, the yield rates (defined as the percentage of deals on the total number of submitted projects) and the role of women and minority entrepreneurs.

In Europe, the European Association for Business Angels (EBAN) keeps the most comprehensive record of visible business angel activity based on the information provided by European business angel networks (hereafter BANs), Federations of BANs, individual business angels and other validated early stage investors which responded to EBAN's Survey.⁹ The Survey is conducted yearly and typically reports activities which have taken place in the previous year. The collection of data is pursued through a web form activated from the EBAN website or directly e-mailed to the Secretariat of EBAN through a pre-formatted survey. Therefore, the figures presented in each report might be not representative of the entire European market, and the number of respondents might differ yearly. Yet, at present data obtained from this survey still represent by far the richest source of information on the business angel activity in the "visible market" (indeed, the Bureau Van Dijk has chosen EBAN as a data partner for the inclusion of business angels and seed funds *deals* information in Zephyr).

Until 2009, EBAN used to make this information publicly available through its yearly document "European directory of business angel networks in Europe", by reporting - where available - aggregate country-specific information of the angel market size (namely, the number of projects submitted to the angels, the number of angels, the number of deals made and sometimes the average amount of the deal) along with a detailed breakdown of the specific activities of each business angel operating in each country. As of 2010, however, EBAN has chosen not to disclose these data any longer and to retrieve the whole set of previously freely available information from its website. Hence, for the countries whose aggregate information prior to 2010 is missing and for updates from 2010, we have hand-collected data on the three dimensions of interest by exploring the institutional websites of each BAN.

⁹Investments that individuals make on their own or with *ad hoc* groups of friends, the so-called "invisible market", are not captured by EBAN's data. On the relevance of the invisible market see OECD (2011).

As a result, our empirical analysis relies on data which, prior to 2010, are no longer available to the general public, and from 2010 are collected via an accurate data research on the institutional websites of each BAN. We have then assembled and created a unique and original dataset of yearly observations across 17 developed countries on 1) the total number of entrepreneurial projects *submitted* to each business angel as a proxy for L_E , 2) the number of business angels as a proxy for L_K , and 3) the number of deals as a proxy for M . We handle an unbalanced panel dataset of EU-15 countries (except Luxembourg and Ireland), plus Norway, Poland, Russia and the US over the period 1996-2014. A summary description of these data is provided in Table 1.

INSERT TABLE 1 HERE

As shown in Table 1, the size of the angel market along our dimensions of interest is remarkably different across countries and, within each country, over time (as shown by the large value of the standard deviation). In particular, the Anglo-Saxon countries (UK and USA) display the largest number of business angels and of deals, followed by the most populous European countries (Germany and France).

Finally, a look at the evolution over time of our data reveals an increasing trend over the observed period in the size of the angel market for the whole sample of countries (particularly pronounced for the European continental countries). To give a rough estimate of this expansion along our time span, we compute the growth rate of each dimension between the first and the last period of observation for each country, and then we take the median values over the whole set of countries. We find that, over the period 1996-2014, the number of submitted projects, business angels and deals in the median country has respectively grown by 91%, 140% and 96%.

3.2 A CES-Type Matching Function

Using the data illustrated above, we carry out pooled and fixed-effect estimations of different specifications of the matching function starting with the logarithmic transformation of the following CES-type function:

$$M_{i,t} = A \left[\delta_E (L_E)_{i,t}^\Psi + \delta_K (L_K)_{i,t}^\Psi \right]^{\frac{1}{\Psi}} \exp(\beta_c \mathbf{c}_{i,t} + \varepsilon_{i,t}), \quad (2)$$

where $M_{i,t}$ is the number of deals in country i at time t ; $(L_E)_{i,t}$ and $(L_K)_{i,t}$ are the number of projects submitted and of business angels in country i at time t , respectively;

$\mathbf{c}_{i,t}$ is a vector of controls; v is the return-to-scale parameter; δ_E and δ_K are share parameters (with $\delta_E + \delta_K \equiv 1$), A is a scale technology parameter. For this function, the Hicks elasticity of substitution between the two input factors is given by $\eta = 1/(1 - \Psi)$. The CES collapses to a Cobb-Douglas function when $\eta \rightarrow 1$ (that is, when $\Psi \rightarrow 0$).

We now test (Hp 1), (Hp 2) and (Hp 3) in specification (2) using the dataset presented in the previous subsection. In particular, the hypothesis of positive marginal returns to both inputs (part (i) of (Hp 1)) requires δ_E, δ_K, v, A being strictly positive, given that $\partial M_{i,t}/\partial (L_j)_{i,t} = Av\delta_j L_j^{\Psi-1} (\delta_j (L_j)_{i,t}^\Psi + \delta_{-j} (L_{-j})_{i,t}^\Psi)^{\frac{v}{\Psi}-1} \exp(\beta_{\mathbf{c}} \mathbf{c}_{i,t} + \varepsilon_{i,t})$ (for part (ii) of (Hp 1) on decreasing marginal returns see below).

(Hp 2) on the existence of a positive technological complementarity across the two inputs is verified when the cross-partial derivative of (2), which is given by

$$\begin{aligned} \frac{\partial M_{i,t}}{\partial (L_j)_{i,t} \partial (L_{-j})_{i,t}} &= Av(v - \Psi) \delta_j \delta_{-j} \left((L_j)_{i,t} (L_{-j})_{i,t} \right)^{\Psi-1} (\delta_j (L_j)_{i,t}^\Psi + \delta_{-j} (L_{-j})_{i,t}^\Psi)^{\frac{v}{\Psi}-2} \\ &\quad \cdot \exp(\beta_{\mathbf{c}} \mathbf{c}_{i,t} + \varepsilon_{i,t}), \end{aligned}$$

is strictly positive. In addition to the parameter constraints stated above, this occurs whenever $(v - \Psi) > 0$. (Hp 3) on the homogeneity of degree one of (2), instead, requires $v = 1$.

Finally, it can be easily demonstrated that positive input complementarity and constant returns to scale imply marginal diminishing returns to both inputs - that is, part (ii) of (Hp 1) is automatically verified whenever $v = 1$ (Hp 3) and $(v - \Psi) > 0$ (Hp 2).¹⁰

The three hypotheses are tested against five specifications of the log-CES function, which differ with respect to the set of controls included in vector $\mathbf{c}_{i,t}$. In model 1, the $\mathbf{c}_{i,t}$ vector includes a time trend only; model 2 adds country group dummies;¹¹ model 3 adds interaction terms between the time trend variable and the country group dummies; model 4 and model 5 replicate models 2 and 3, respectively, but replace

¹⁰In fact, on the one hand, $v = 1$ and $(v - \Psi) > 0$ imply $\Psi < 1$. On the other hand, after a few algebraic steps we can write $\partial^2 M_{i,t}/\partial (L_j)_{i,t}^2 = Av\delta_j L_j^{\Psi-2} \cdot (\delta_j (L_j)_{i,t}^\Psi + \delta_{-j} (L_{-j})_{i,t}^\Psi)^{\frac{v}{\Psi}-2} \left[(\Psi - 1) \cdot \delta_{-j} (L_{-j})_{i,t}^\Psi + (v - 1) \cdot \delta_j (L_j)_{i,t}^\Psi \right] \exp(\beta_{\mathbf{c}} \mathbf{c}_{i,t} + \varepsilon_{i,t})$ which is strictly negative whenever $\Psi < 1$.

¹¹Countries have been aggregated in the following five groups: 1) Continental European countries (Austria, Belgium, Germany and the Netherlands); 2) Mediterranean European countries (France, Greece, Italy, Portugal and Spain); 3) Scandinavian countries (Denmark, Finland, Norway and Sweden); 4) Eastern European countries (Poland and Russia); 5) Anglosaxon countries (United Kingdom and the United States).

country group dummies with country dummies. Including interaction terms between country identifiers and the time trend serves to capture the changing role of country-specific characteristics on the observed evolution of the matching function in presence of an unbalanced panel dataset.

The results from the nonlinear estimations of the log-CES matching function are shown in Table 2.¹² Estimates are in line with the theoretical hypotheses. In particular, in all specifications (i) parameters δ_E, δ_K, v, A are strictly positive and the joint test for the null hypothesis that δ_E, δ_K, v, A are jointly null is strongly rejected (as requested by part (i) of (Hp 1)); (ii) v is significantly positive and around 1, Ψ is always not significantly different from zero, and the null hypothesis of the joint test for the null hypothesis that $\delta_E, \delta_K, v, A, (v - \Psi)$ are jointly null is strongly rejected (in line with (Hp 2)); (iii) the null hypothesis on the F-test on $v = 1$ (that is, (Hp 3) on constant returns to scale) cannot be rejected. As claimed above, (Hp 2) and (Hp 3) imply part (ii) of (Hp 1).¹³

INSERT TABLE 2 HERE

Finally, with a Ψ not significantly different from zero, we do not reject the hypothesis of a unitary elasticity of substitution between the two inputs. This induces us to consider, in the next subsection, a Cobb-Douglas specification of the matching function.

3.3 A Cobb-Douglas Matching Function

Given a Cobb-Douglas matching function of the form $M_{it} = A (L_E)_{i,t}^{\beta_E} (L_K)_{i,t}^{\beta_K} \exp(\beta_{\mathbf{c}} \mathbf{c}_{i,t} + \varepsilon_{i,t})$, we now estimate the following log-transformation:

$$m_{i,t} = \beta_0 + \beta_E (l_E)_{i,t} + \beta_K (l_K)_{i,t} + \beta_{\mathbf{c}} \mathbf{c}_{i,t} + \varepsilon_{i,t} \quad (3)$$

where $m_{i,t}$ is the log of the number of deals in country i at time t ; $(l_E)_{i,t}$ and $(l_K)_{i,t}$ are the logs of the number of projects submitted and of the business angels in country i at time t , respectively; $\mathbf{c}_{i,t}$ is a vector of controls. Taken together, part (i) of (Hp 1) and

¹²Estimates are computed using nonlinear least squares, and the residuals have an approximately normal distribution.

¹³In addition, the joint test for the null hypothesis that δ_E, δ_K, v, A are null and $\psi = 1$ is strongly rejected in all specifications.

(Hp 2) require $\beta_E, \beta_K, A > 0$, part (ii) of (Hp 1) requires β_E, β_K being jointly lower than 1, while (Hp 3) on constant returns to scale implies $\beta_E + \beta_K = 1$.

Function (3) is estimated via a *robust* regression approach to deal with the presence, in the dataset, of outliers that can distort the ordinary least squares estimator (OLS). By considering squared residuals, OLS tend to give greater importance to observations with very large residuals and, consequently, distort the parameters' estimation in presence of outliers. Adopting the graphical tool proposed by Rousseeuw and Van Zomeren (1990), Figure 1 shows that several outliers are present, suggesting that there is a serious risk that the OLS estimator be strongly attracted by outliers (Rousseeuw and Leroy, 1987).¹⁴ To tackle this issue, and following the traditional related literature (Hamilton, 1991), we adopt a Huber's monotonic M-estimator, which combines the M-estimator with an initial step that removes high-leverage outliers, based on Cook's distance. In particular, the adopted estimator takes the Cook's D for each observation by first running an OLS regression (Cook's distance is a way to estimate the influence of a data point when performing a least-squares regression analysis). Then the procedure drops observations with Cook's distance greater than 1 and proceeds with iteratively re-weighted least squares, with each observation being assigned higher weights, the smaller the computed residuals are. In order to improve the estimator efficiency, two types of weighting procedure are used: Huber weighting and biweighting. The iterating procedure stops until the estimated coefficients converge, that is, until the maximum change between the weights from one iteration to the next is below tolerance.

INSERT FIGURE 1 HERE

We estimate three specifications of (3) which, as in the previous model, differ according to the set of variables included in the vector of controls $\mathbf{c}_{i,t}$. In model 1, the $\mathbf{c}_{i,t}$ vector includes a time trend variable only; model 2 adds year and country dummies; model 3 adds to model 1 country-specific time trends.

¹⁴In particular, two observations for Belgium and Norway are *bad leverage points*, meaning that their explanatory variables are slightly different from those of the rest of data and their outcomes are higher than they should be according to the fitted model. The collected data for US are large *good leverage points*, suggesting that the characteristics of the US business angels market are rather different from the other countries but that the number of deals is consistent with what the model predicts. Finally, few other observations (i.e., for Italy, Portugal, Denmark, Netherlands and Poland) are *vertical outliers*, being standard in their characteristics but more or less successful in terms of number of deals than the model would suggest.

Estimation results are shown in the first three columns of Table 3 and are in line with theoretical predictions. In particular, (Hp 1) and (Hp 2) are not rejected since the share parameters are jointly significantly positive and different from one: a 1% increase in the number of submitted projects (business angels) leads to positive increase in the number of deals by between 0.37% (0.38%) and 0.64% (0.60%). The F-test on the parameter restriction $\beta_E + \beta_K = 1$ does not reject the hypothesis of constant returns to scale (Hp 3).

INSERT TABLE 3 HERE

3.4 A Translog Matching Function

By definition, a Cobb-Douglas matching function assumes that the elasticity of output with respect to each input is constant, that is, that $\partial m_{i,t} / \partial (l_j)_{i,t} = \beta_j$. In this section, we consider a generalization of model (3) - the so called transcendental logarithmic (or simply, translog) matching function - in which the output elasticity is allowed to vary with the values of both inputs.¹⁵

The translog function can be written as

$$m_{i,t} = \beta_0 + \beta_E (l_E)_{i,t} + \beta_K (l_K)_{i,t} + \beta_{EK} \left[(l_E)_{i,t} \cdot (l_K)_{i,t} \right] + \quad (4)$$

$$+ \beta_{EE} \left[(l_E)_{i,t} \right]^2 + \beta_{KK} \left[(l_K)_{i,t} \right]^2 + \beta_{\mathbf{c}} \mathbf{c}_{i,t} + \varepsilon_{i,t}.$$

Compared to the Cobb-Douglas, the translog adds (i) a log-interaction term between the two inputs, in order to verify the existence of input complementarity not only in levels but in *elasticities* (in which case β_{EK} would be significantly positive) and (ii) the squares of the two log-inputs to verify whether the elasticity of each input is decreasing with the log-value of that input (that is, whether $\beta_{EE}, \beta_{KK} < 0$). The $\mathbf{c}_{i,t}$ vector includes country dummies and their interaction terms with the time trend variable.

The scale elasticity of a translog is defined by $\epsilon = \epsilon_E + \epsilon_K$, where $\epsilon_E = \beta_E + \beta_{EK} (l_K)_{it} + 2\beta_{EE} (l_E)_{it}$ is the elasticity of new deals with respect to the number of submitted projects, and $\epsilon_K = \beta_K + \beta_{EK} (l_E)_{it} + 2\beta_{KK} (l_K)_{it}$ is the elasticity of new deals with respect to the number of BAs. Hence, and differently from the Cobb-Douglas case, input elasticities are not assumed constant but depend on the scales of

¹⁵Useful references for the translog are, among others, Warren (1996), Yashiv (2000) and Kangasharju et al. (2005).

both inputs. In the estimation, as usual for the translog function, input elasticities and returns to scale will be evaluated at the sample medians of l_E and l_K . Theoretical hypotheses can be summarized as follows. Part (i) of (Hp 1) requires $\epsilon_E, \epsilon_K > 0$; part (ii) of (Hp 1) needs $\beta_{EE}, \beta_{KK} < 0$; (Hp 2) is verified when $\beta_{EK} > 0$; finally, (Hp 3) requires $\epsilon = 1$.

Before moving to the empirical findings, note that the three models (2), (3) and (4) are all closely related to each other. The log-linear Cobb-Douglas matching function (3) is clearly nested into the translog specification (4) (and thus obtainable from the latter imposing the following restriction: $\beta_{EK} = \beta_{EE} = \beta_{KK} = 0$). On the other hand, the translog specification can be obtained from a second-order Taylor approximation of the logarithmic transformation of the CES specification (2).¹⁶

Also model (4) is estimated via robust regression. Results are shown in column (4) of Table 3. The estimated coefficients of the matching function all have the expected signs and are highly statistically significant. In particular, the log-interaction term (β_{EK}) is significantly positive, meaning that the impact of a 1% increase in one input (the number of business angels or the number of submitted projects) is increasing in the log-value of the other input. On the other hand, the coefficients on the squared log-inputs (β_{EE}, β_{KK}) are significantly negative and jointly different from zero, suggesting that the impact of a 1% increase in one input is decreasing in the log-value of that input.

The two input elasticities, ϵ_E and ϵ_K , calculated at the sample median of the explanatory variables l_E, l_K , are jointly significantly positive and equal to 0.7175 and 0.2162, respectively (that is, respectively higher and lower than those obtained under the Cobb-Douglas estimation). The resulting scale elasticity of the matching function, $\epsilon = \epsilon_E + \epsilon_K$, is then equal to 0.9337. As with the previous two model specifications, even in this case the null hypothesis of constant returns to scale ($\epsilon = 1$) cannot be rejected.

3.5 Robustness Analysis

From a theoretical standpoint, our previous estimates of the matching function may suffer from two possible sources of endogeneity. The first is the presence of *unobservable*

¹⁶More precisely, when the elasticity of substitution is *in the neighborhood* of unity, a two-input log-CES function may be approximated by a Taylor expansion which has the form of (4) under the following restrictions: $\beta_{EK} = -2\beta_{EE} = -2\beta_{KK}$ (Kmenta, 1967).

shocks affecting the decision to finance a project (that is, the matching process within each business angel) and the number of agents on either side of the market (namely, the number of business angels and/or the number of submitted projects). For example, the number of projects submitted to a business angel is very likely to depend on the efficiency of the matching process (which in turn depends on such fundamental characteristics as the quality of institutions regulating the market). Therefore, random shocks to matching efficiency affect the number of matches both directly through the matching technology and indirectly through entrepreneurs' behavior. In this case, the number of projects submitted could be endogenous, and our previous estimation strategy would fail to account for that endogeneity (Borowczyk-Martins et Al., 2013).

The second source of endogeneity is due to the so called problem of *time aggregation* (Burdett et Al., 1994, Petrongolo and Pissarides, 2001). This problem arises when flow variables are estimated as functions of stock variables. The matching process is a dynamic one, implying that the number of submitted projects is depleted by the number of deals. However, since the number of deals is measured as a flow over a time period and the number of submitted projects as a stock *at the beginning* of the same period, a *downward bias* in the estimated elasticity of deals to the number of submitted projects occurs. Note that this bias is not likely to occur when estimating the impact of the number of business angels on matches since angels can finance more than one project at time.¹⁷

In order to control for the previous sources of endogeneity, we employ two different methods. The first "augments" the Cobb-Douglas and the translog model with a set of relevant macroeconomic controls. The second runs a system-GMM estimator on both models. We consider them in order.

The best strategy to deal with possible unobservable macroeconomic shocks would be to include country-specific year dummies instead of country-specific time trend.

¹⁷Due to the peculiar nature of our data, a further source of endogeneity typically arising in the estimation of the matching function, the so-called *spatial aggregation* problem, is not an issue in our setting. In traditional labor market matching functions, the spatial aggregation bias, for example at a country level, arises as a consequence of neglected interactions between vacancies and unemployed across regions on the number of matches (Coles and Smith, 1996). By considering *angel-specific applications*, and not just the number of financing seekers, however, we explicitly take into account the possibility that an entrepreneur looks for multiple sources of finance by submitting her project to several business angels, possibly localized in different regions within the same country or in different countries. Hence, our data already incorporate the whole set of interactions between entrepreneurs and business angels in affecting the number of matches.

However, due to the small number of observations, the whole set of interactions between country dummies and year dummies would leave the model with too few degrees of freedom to test the variables. This implies that our models (3) and (4) are equipped to control for country-specific invariant unobservables and time-variant country-specific unobservables but not for country-specific shocks which may have occurred between 1996 and 2014. Hence, to proxy for country-specific macroeconomic changes which might affect the matching process as well as the input variables over the years, we "augment" both models with relevant macroeconomic indicators. In particular, we consider the following variables: the amount of domestic credit to private sector provided by banks and that provided by the financial sector, both as a percentage of GDP (The World Bank database), the rate of GDP growth (The World Bank database), the level of GDP per capita (The World Bank database) and the amount of funds venture capitalists invested during the year (sources NVCA for the US, EVCA for Europe, RVCA for Russian Federation).

The estimation results of the augmented Cobb-Douglas model and of the augmented Translog model are shown in Table 3 (Columns 5 and 6). Estimated coefficients are still statistically significant and consistent with the theoretical hypotheses. For the Cobb-Douglas matching function (Column 5), the magnitudes of the share parameters are slightly different. In particular, β_E jumps to 0.54% (from the 0.37%, obtained under the same model but without controls). β_K drops to 0.46% (from the 0.60% still obtained under the same model but without controls). On the other hand, the two input elasticities ϵ_E and ϵ_K , calculated at the sample medians of the explanatory variables l_E, l_K , are very close to those obtained in the corresponding estimation without controls, that is, $\epsilon_E = 0.72$ and $\epsilon_K = 0.22$ (Column 6).

As a second strategy to cope with the problem of endogeneity of the two inputs, we adopt the so-called system-GMM estimator on our two models, both with and without additional controls. Following Blundell and Bond (1998), the choice of employing a system-GMM estimator is motivated by the large finite sample biases due to the time series persistence properties of some of the variables (in particular, the number of business angels). This estimator considers a system of equations formed by the equation in first-differences (to eliminate unobserved firm-specific effects) and the equation in levels. The instruments used for the equation in first-differences are appropriate lags of the explanatory variables in levels (as in a standard panel first differenced GMM); predetermined and endogenous variables in levels are instead instrumented with their lagged first differences. The main assumption here is that the country-specific effects

are uncorrelated with disturbance-term changes. We estimate a number of different specifications using the explanatory variables from the current period and with lags, treating them as either predetermined or endogenous.

The system panel results are reported in Table 4. In the first three columns, we treat all explanatory variables as predetermined. In order to prevent that a large number of instruments overfits the instrumented variables biasing the results towards those of robust regressions, we limit the set of instrumental variables to: (i) the lagged levels of the explanatory variables dated $(t - 1)$ and $(t - 2)$ and the lagged value of the dependent variable dated $(t - 2)$ in the first-differenced equations, and (ii) the corresponding lagged first-differences dated $(t - 1)$ plus year dummies in the levels equations. In the last three columns, we treat all explanatory variables as endogenous. Again, in order to limit the number of instruments, the set of instrumental variables for the first-differenced equations include the lagged levels of the explanatory variables dated $(t - 2)$ and $(t - 3)$. The set of instrumental variables for the levels equations include the lagged first-differences of the same variables dated $(t - 1)$ plus year dummies.

INSERT TABLE 4 HERE

The results from the System-GMM estimations confirm the theoretical assumptions. In all specifications, the elasticities of deals to both input shares are significantly positive and decreasing with the log-value of each input. Moreover, inputs are complementary. Finally, although the magnitudes of estimated coefficients differ across specifications, the null hypothesis of constant returns to scale is always not rejected.

In line with the matching function literature cited above, our system-GMM results confirm the presence of a possible time aggregation (downward) bias in the estimated value of β_E under the baseline robust regression of the Cobb-Douglas matching function specification. Table 5 summarizes the estimated input elasticities of both models (Cobb-Douglas and translog) under the three different estimation methods utilized: robust regression, system-GMM with explanatory variables treated as predetermined and system-GMM with explanatory variables treated as endogenous. In the Cobb-Douglas model, the estimated elasticity of matches to the number of submitted projects increases from 0.5 under the robust regression to a number close to 1 under the system-GMM, while at the same time β_K almost halves. In the translog model, the values of

two input elasticities, calculated at the sample median of the explanatory variables, are more stable across the different estimation methods.¹⁸

INSERT TABLE 5 HERE

The consistency of the GMM estimator depends on the validity of the instruments, which we test using two specification tests. The first is the standard Sargan test of overidentifying restrictions, which tests the validity of the instruments by analyzing the sample analog of the moment conditions used in the estimation process. The second test examines the hypothesis that there is no second-order serial correlation in the first-differenced residuals. The panel system-GMM estimates pass the specification tests. According to the Sargan statistic test for overidentifying restrictions reported at the bottom of Table 4, the set of instruments are valid, and the Arellano-Bond test for AR(2) implies that the error terms in levels are not serially correlated.

4 Concluding Remarks

This paper has described the market for entrepreneurial finance as a decentralized market in which entrepreneurs and financiers search and match with each other. After introducing a standard aggregate matching technology to capture such relationship, we have specified the most common hypotheses imposed in the literature (and in Appendix A we have explicitly demonstrated that such hypotheses have important theoretical and policy implications). Using a new, hand-collected dataset on the business angel market, we have then estimated and compared three different specifications of the matching function (CES, Cobb-Douglas and Translog), and we have verified empirically the previously stated hypotheses. Across all model specifications, the estimated matching function exhibits positive and decreasing marginal productivities of its two inputs, a positive degree of technological complementarity between them and constant returns to scale.

We wish to conclude this paper putting our main findings in a policy perspective. The role of public policy in fostering entrepreneurial activity is a rather controversial

¹⁸This is not surprising given that, differently from the Cobb-Douglas model, in the translog model the original downward bias in the estimation of ϵ_E is mitigated by the presence of the initial value of l_E inside the expression for ϵ_E .

issue. At one end of the spectrum, policy supporters claim that, behind every successful story of entrepreneurial innovation (from the Silicon Valley to the Singapore venture capital industry), the role of public policy has always been crucial, especially at the very early stages of development. In the words of Lerner (2010, p.42), "every hub of cutting-edge entrepreneurial activity in the world today had its origins in proactive government intervention".¹⁹ At the other end, policy detractors tend to emphasize the limits of policy intervention -for instance, in terms of the government's incompetence or capture by special interests- and provide evidence of failed and expensive measures of public stimulus to private entrepreneurship (the so called "boulevard of broken dreams", Lerner, 2010).

As already mentioned in the Introduction, this policy debate is partly inspired by the theoretical literature. For instance, models admitting multiple equilibria provide a powerful narrative behind the need for policy intervention: as equilibrium multiplicity implies the existence of coordination failures -that is, of equilibria characterized by sub-optimally low paces of economic activity-, government intervention may serve to spur a *virtuous* cycle, that is, to favor the coordination of economic agents towards a superior equilibrium configuration. The evidence on the constant returns to scale of the matching process produced in this paper challenges this narrative. Hence, this paper adds a note of prudence in evaluating the opportunity of public efforts to boost entrepreneurship in this peculiar market. Needless to say, this evidence is far from conclusive in any dimension, and further work remains to be done in this as well as in the other relevant markets for entrepreneurial finance.

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A The Theoretical Framework

The model we introduce in this section is a simple, dynamic model of search and matching between entrepreneurs and financiers. The basic framework introduced in Subsection A1 transposes the classical Diamond's (1982) *cocunut* model to the entrepreneurial finance market and extends it to a two-sided search structure. Subsection A2 discusses the issue of equilibrium multiplicity inside this framework.

A.1 The Fair of Innovation

The world is populated by E entrepreneurs and K financiers who must decide whether to participate or not in a "fair" of new ideas. New ideas arrive randomly to the entrepreneurs according to a Poisson process with parameter σ . However, in order for such ideas to become marketable innovations, entrepreneurs need to meet financiers and convince them about their profitability. This process of search and matching occurs inside the fair. Hence, the model revolves around the entry decisions of these two types of agents, which depend on the costs and benefits of attending the fair.

For entrepreneurs each new idea is associated with an entry cost, denoted by c_E , which is idiosyncratically drawn from a c.d.f. $F(c_E)$ in the support $[0, \bar{c}_E]$. Financiers' entry costs, denoted by c_K , are instead distributed according to a c.d.f. $G(c_K)$ in the support $[0, \bar{c}_K]$. c_E and c_K may also be thought of as inversely capturing, respectively, the quality of the new idea and the managerial talent of the financier.

The expected benefit for entrepreneurs (financiers) is given by the chance of matching the right financier (entrepreneur) -and, hence, of transforming the new idea into a profitable firm-, multiplied by the total profits associated with the new entrepreneurial venture. This expected benefit is obtained via a standard dynamic programming argument. In particular, we determine the values of being *inside* and *outside* the fair for each type of agent, so that the expected benefit from fair participation is given by the difference between these two values.

The value of being outside the fair, respectively denoted by V_E^0 for entrepreneurs and V_K^0 for financiers, is defined by the two following asset equations:

$$rV_E^0 = \sigma \int_0^{c_E^*} (V_E^1 - V_E^0 - c_E) dF(c_E), \quad (\text{A1})$$

$$rV_K^0 = V_K^1 - V_K^0 - c_K,$$

where r is the exogenous riskless interest rate, and V_E^1, V_K^1 denote the values of being inside the fair for entrepreneurs and financiers, respectively defined by²⁰

$$rV_E^1 = \alpha_E \theta \pi + V_E^0 - V_E^1, \quad (\text{A2})$$

$$rV_K^1 = \alpha_K (1 - \theta) \pi + V_K^0 - V_K^1.$$

In the two equations above, π is total instantaneous innovation profits, $\theta \in (0, 1)$ is the entrepreneurs' share of these profits, and α_E, α_K denote the instantaneous probabilities of matching for, respectively, entrepreneurs and financiers attending the fair. The production process of new entrepreneurial ventures is described by the aggregate matching function (1). Hence, the instantaneous probabilities of matching are given by $\alpha_E = M/L_E$ and $\alpha_K = M/L_K$, where L_E, L_K denote, respectively, the (endogenous) stocks of entrepreneurs and financiers currently inside the fair.

The previous equations have the usual interpretations. Equation (A1) captures the entrepreneurs' return from being outside the fair as the instantaneous probability of a

²⁰Given our exclusive focus on the steady state equilibrium of the model, we have imposed $\dot{V}_j^h = 0$ for $h = 0, 1$ and $j = E, K$.

new idea times the corresponding payoff, which is given by the capital gain associated with participating in the fair minus the entry cost. Equation (A2) represents the entrepreneurs' return from being inside the fair as the chance of matching the right financier times the share of innovation profits, plus the capital gain or loss associated with exiting from the fair. An analogous interpretation holds for the financiers' equations.²¹

As a result, the benefit from *entering* into the fair is given by the difference $V_E^1 - V_E^0$ for entrepreneurs and $V_K^1 - V_K^0$ for financiers. In equilibrium, there exist an inframarginal entrepreneur and an inframarginal financier for whom entry cost equalizes entry benefit, that is, $c_j^* = V_j^1 - V_j^0$ for $j = E, K$. After some elementary algebra, we obtain the two optimal entry conditions as

$$c_E^* = \frac{\alpha_E \theta \pi + \sigma \int_0^{c_E^*} c_E dF(c_E)}{1 + r + \sigma F(c_E^*)} \quad (\text{A3})$$

$$c_K^* = \frac{\alpha_K (1 - \theta) \pi}{1 + r} \quad (\text{A4})$$

linking the threshold cost c_j^* to the probability of successful matching α_j (with $j = E, K$).²²

A.2 Coordination Failures in the Financial Market of Innovation

We are now ready to characterize the stationary equilibrium/equilibria of the model. Along the steady state, the inflows and outflows from the fair of innovation must be equal for both entrepreneurs and financiers, that is

$$\dot{L}_E = \sigma (E - L_E) F(c_E^*) - L_E \cdot \alpha_E = 0, \quad (\text{A5})$$

and

$$\dot{L}_K = (K - L_K) G(c_K^*) - L_K \cdot \alpha_K = 0, \quad (\text{A6})$$

²¹The difference between the entrepreneurs' and the financiers' pairs of equations is simply due to the fact that, when assessing the value of being outside the fair, each entrepreneur does not know the value of her idea (because she is waiting for one), while each financier knows her talent (which is time-invariant).

²²The relation is obviously positive: an increase in the probability of a successful matching (α_j) leads to an increase in the cutoff value of the entry cost c_j^* .

where $L_E \cdot \alpha_E = L_K \cdot \alpha_K = M$. The two equations above describe, respectively, the evolution of entrepreneurs and financiers over time: along the steady state, the number of entrepreneurs deciding to participate in the fair ($\sigma(E - L_E)F(c_E^*)$) must equalize the number of entrepreneurs who have successfully matched with financiers and have thus exited from the fair ($L_E \cdot \alpha_E$) (an analogous interpretation can be given to the second equation).

A stationary equilibrium for this economy is defined as any 4-tuple (L_E, L_K, c_E^*, c_K^*) that solves the four equations (A3), (A4), (A5) and (A6). In search and matching models, the possibility of multiple equilibria is due to the presence of a thick market externality (whereby the entrepreneurs' payoff from market participation is increasing in the number of financiers, and *viceversa*). The next proposition formally links the number of stationary equilibria to the returns to scale of the matching function (which govern the strength of this externality).

Proposition 1 *If the matching function (1) is homogeneous of degree 1, the economy admits one and only one stationary equilibrium.*

Proof. First pose $\Omega \equiv L_K/L_E$. Given that (1) has CRS, we can write $\alpha_E \equiv M/L_E = m(\Omega)$, and $\alpha_K \equiv M/L_K = (1/\Omega)m(\Omega)$. The entry conditions, (A3) and (A4), are then both functions of Ω only, the former increasing, the latter decreasing, that is, $c_E^* \left(\overset{+}{\Omega} \right)$ and $c_K^* \left(\overset{-}{\Omega} \right)$. By substituting these functions respectively into (A5) and (A6), we obtain

$$L_E = \frac{\sigma EF(c_E^*(\Omega))}{1 + \sigma F(c_E^*(\Omega))} \quad (\text{A7})$$

and

$$L_K = \frac{KG(c_K^*(\Omega))}{1 + G(c_K^*(\Omega))}. \quad (\text{A8})$$

Standard differential calculus proves that $L_E(\Omega)$ defined in (A7) is monotone increasing in Ω , while $L_K(\Omega)$ defined in (A8) is monotone decreasing in Ω . Hence, the function defined as their ratio, $L_K/L_E(\Omega)$, is unambiguously decreasing in Ω . Given that it is $\Omega \equiv L_K/L_E$, a stationary equilibrium is a fixed point of this function. We now prove that this function admits one and only one fixed point. Define $g(\Omega) \equiv L_K/L_E(\Omega) - \Omega$. There exist sufficiently low values of Ω such that $g(\Omega) > 0$, as well as sufficiently high values of Ω such that $g(\Omega) < 0$.²³ Given that $g(\Omega)$ is a continuous and monotone

²³The standard assumptions on the matching function imply that

$$\lim_{\Omega \rightarrow 0} \frac{L_K}{L_E}(\Omega) = +\infty$$

decreasing function in Ω , the intermediate value theorem guarantees the existence of one and only one Ω^* such that $g(\Omega^*) = 0$, that is, such that $L_K/L_E(\Omega^*) = \Omega^*$. Finally, it might still be the case that multiple equilibria exist, even though they are all characterized by a unique ratio Ω^* . This instance, however, can be excluded once we realize that $L_E(\Omega)$ and $L_K(\Omega)$, defined in (A7) and (A8), are monotone functions of Ω . ■

How do we interpret equilibrium multiplicity? For the sake of illustration, suppose that our economy admits two (non-degenerate) equilibria, respectively denoted by superscripts O, P , with $(L_j)^O > (L_j)^P$ for $j = E, K$ (an example of this kind is developed at the end of this section). These two equilibria can be interpreted as self-fulfilling equilibria triggered, respectively, by optimistic or pessimistic expectations. Whenever entrepreneurs expect a high number of financiers to be matched with $(L_K^e = (L_K)^O)$ where the superscript e stands for "expected", their number will be high as well, $(L_E)^O$. Similarly, whenever financiers expect a high number of entrepreneurs $(L_E^e = (L_E)^O)$, their number will also be high, $(L_K)^O$. Equilibrium O can be labelled as the optimistic (or thick) equilibrium. Via a totally symmetric argument, expecting few entrepreneurs and financiers entering into the market makes the agents converge towards the low-entry equilibrium P , which can be referred to as the pessimistic (or thin) equilibrium.

Given that in our model only profitable innovations are pursued, whenever multiple equilibria exist, they can be Pareto-ordered from the lowest to the highest number of innovations (matches) produced by the economy. Welfare is thus maximized at the equilibrium characterized by the highest number of matches: all other equilibria are sub-optimal and are the result of a *coordination failure* between entrepreneurs and financiers. In this respect, the model suggests that *animal spirits* matter in the process of innovation, in the sense that, whether a high or a low activity equilibrium is reached may depend on a self-fulfilling mechanism triggered by entrepreneurs' and financiers' expectations. Usually, this line of argument provides the main theoretical justification of policy intervention. We further discuss this issue in the concluding remarks.

Example. Consider the model developed in the previous sections and further suppose that (i) the matching function (1) is Cobb-Douglas with increasing returns

and

$$\lim_{\Omega \rightarrow +\infty} \frac{L_K}{L_E}(\Omega) = 0.$$

Even though they are not necessary, these two results ensure the existence of the two regions where $g(\Omega) > 0$ and $g(\Omega) < 0$.

to scale: $M = AL_E^{\beta_E} L_K^{\beta_K}$ with $\delta \in R_+$, $\beta_E, \beta_K < 1$ and $\beta_E + \beta_K > 1$, and that (ii) entry costs are the same for every entrepreneur and every financier, \underline{c}_E and \underline{c}_K .²⁴ This economy admits three stationary equilibria. The first (thin) equilibrium is given by the pair $\left((L_E)^P, (L_K)^P\right)$ that solves the following system:²⁵

$$\begin{cases} \underline{c}_E = \frac{\frac{\delta L_E^{\beta_E} L_K^{\beta_K}}{L_E} \theta \pi + \sigma \underline{c}_E}{1+r+\sigma} \\ \underline{c}_K = \frac{\frac{\delta L_E^{\beta_E} L_K^{\beta_K}}{L_K} (1-\theta) \pi}{1+r}. \end{cases} \quad (\text{A9})$$

The second (thick) equilibrium is instead given by the pair $\left((L_E)^O, (L_K)^O\right)$ that maximizes market entry and thus the size of the innovation fair:

$$\begin{cases} (L_E)^O = \sigma \left(E - (L_E)^O \right) \\ (L_K)^O = K - (L_K)^O. \end{cases}$$

Finally, the third (degenerate) equilibrium is a sort of "no-innovation trap" and is given by $\left((L_E)^T, (L_K)^T\right) = (0, 0)$. It is possible to prove that the trap and the thick equilibrium are stable, while the thin equilibrium is unstable.²⁶

²⁴In some respects, this example resembles the one developed by Diamond (1982) in Section IX.

²⁵Under constant returns to scale, this system is impossible (as it is made up of two equations in one unknown, L_K/L_E), and hence this equilibrium disappears.

²⁶The two best response functions in (A9) are in fact strictly convex whenever the matching function is characterized by increasing returns to scale. The algebraic proof of this statement, as well as a graphical intuition of the three equilibria, are available upon request from the authors. We omit them for brevity.

Table 1. Summary Statistics.

	No. Deals		No. Angels		No. Projects		Obs.
	mean	sd	mean	sd	mean	sd	
Austria	5	3	71	40	66	21	11
Belgium	36	10	161	135	222	79	8
Denmark	17	29	76	86	38	30	4
Finland	8	6	210	152	36	13	9
France	205	69	2,504	1,004	827	265	4
Germany	36	8	580	174	2,309	889	6
Greece	1	1	11	3	8	4	5
Italy	174	146	262	32	857	625	8
Netherlands	50	24	196	189	178	75	8
Norway	3	1	133	38	37	12	3
Poland	4	2	56	28	100	83	4
Portugal	1	-	18	6	5	1	4
Russia	3	1	88	46	35	7	2
Spain	16	10	273	197	280	181	8
Sweden	43	28	284	157	358	306	4
United Kingdom	245	100	4,959	465	558	282	4
USA	56,586	11,205	257,802	36,919	363,341	96,498	13
Total	7,050	19,094	32,352	86,079	45,318	124,513	105

Table 2. Results from the nonlinear estimation of the log-CES matching function.

	(1)	(2)	(3)	(4)	(5)
logA	-3.5241*** (0.4215)	-3.5375*** (0.6246)	-2.2427*** (0.8482)	-5.0414 (1.5066)	-2.6440 (1.5569)
δ_E	0.7296*** (0.1527)	0.6205*** (0.1753)	0.5615*** (0.1820)	0.7738*** (0.1340)	0.8879* (0.4170)
v	1.1263*** (0.0612)	1.1723*** (0.1278)	1.1407*** (0.1294)	1.5249*** (0.4134)	0.9450* (0.3590)
Ψ	0.3765 (0.8937)	0.3332 (0.7561)	-0.1658 (0.7833)	-0.4123 (0.7686)	-2.1804 (5.8105)
Time trend	YES	YES	YES	YES	YES
Country group dummies	NO	YES	YES	NO	NO
Country group specific time trend	NO	NO	YES	NO	NO
Country dummies	NO	NO	NO	YES	YES
Country specific time trend	NO	NO	NO	NO	YES
N	105	105	105	105	105
R-squared	0.84	0.85	0.86	0.89	0.93
CRS p-value	0.1076	0.3925	0.4809	0.4062	0.3658
$\Psi \rightarrow 0$ p-value	0.6745	0.6604	0.8329	0.5931	0.7265

*** p<0.01, ** p<0.05, * p<0.1

Table 3. Results from the robust estimations of Cobb-Douglas and Translog matching function specifications.

	(1)	(2)	(3)	(4)	(5)	(6)
β_K	0.3834*** (0.0782)	0.3801** (0.1511)	0.5983*** (0.1739)	0.5078*** (0.1345)	0.4601*** (0.1585)	0.5993*** (0.1355)
β_E	0.6414*** (0.0695)	0.5950*** (0.0875)	0.3726*** (0.0886)	1.3356*** (0.0553)	0.5408*** (0.0975)	1.2052*** (0.0521)
β_{EK}				0.0612** (0.0266)		0.1699*** (0.0254)
β_{KK}				-0.0547*** (0.0184)		-0.1143*** (0.0190)
β_{EE}				-0.0852*** (0.0118)		-0.1296*** (0.0108)
Other controls	NO	NO	NO	NO	YES	YES
Time trend	YES	NO	YES	YES	YES	YES
Year dummies	NO	YES	NO	NO	NO	NO
Country dummies	NO	YES	YES	YES	YES	YES
Country specific time trend	NO	NO	YES	YES	YES	YES
N	105	105	105	105	105	105
R-squared	0.945	0.986	0.995	0.999	0.987	0.999
RTS	1.02	0.98	0.97	0.93	1.00	0.93
CRS p-value	0.4266	0.8426	0.8467	0.1103	0.9942	0.1208

*** p<0.01, ** p<0.05, * p<0.1

Table 4. Results from the system-GMM estimations of the Cobb-Douglas and Translog matching function specifications.

	(1) (t-1)(t-2)	(2) (t-1)(t-2)	(3) (t-1)(t-2)	(4) (t-2)(t-3)	(5) (t-2)(t-3)	(6) (t-2)(t-3)
β_K	0.2452* (0.1387)	0.2310* (0.1406)	0.5331*** (0.1915)	0.3269** (0.1639)	0.2154*** (0.0729)	0.9695*** (0.0609)
β_E	0.9062*** (0.0271)	0.9059*** (0.2574)	1.9245*** (0.1779)	0.8090*** (0.0554)	1.0595*** (0.1113)	1.7279*** (0.5042)
β_{EK}			0.1608*** (0.0344)			0.0492** (0.0215)
β_{KK}			-0.0818*** (0.0089)			-0.0544*** (0.0070)
β_{EE}			-0.1953*** (0.0059)			-0.1292* (0.0709)
Other controls	NO	YES	YES	NO	YES	YES
N	105	105	105	105	105	105
RTS	1.15	1.14	1.10	1.14	1.26	1.12
CRS p-value	0.1751	0.5165	0.1383	0.2106	0.1504	0.1089
Sargan Test p-value	0.316	0.125	0.379	0.173	0.251	0.877
A-B test for AR(1) p-value	0.170	0.160	0.157	0.171	0.168	0.167
A-B test for AR(2) p-value	0.268	0.282	0.369	0.265	0.253	0.314

*** p<0.01, ** p<0.05, * p<0.1

Note: Clustered standard errors. The set of additional controls include: the rate of GDP growth (The World Bank database), the value of venture capital investments over GDP (EVCA, NVCA databases), domestic credit to private sector provided by banks over GDP (The World Bank database), domestic credit to private sector provided by financial sector over GDP (The World Bank database), the log value of GDP per capita (The World Bank database).

For estimations (1)-(2)-(3), the set of instruments includes the lagged levels of independent variables dated (t - 1) and (t - 2) and the lagged value of the dependent variable dated (t - 2) in the first-differenced equations and the corresponding lagged first-differences dated (t - 1) plus year dummies in the levels equations.

For estimations (4)-(5)-(6), the set of instruments includes the lagged levels of independent variables and the lagged value of the dependent variable dated (t - 2) and (t - 3) in the first-differenced equations and the corresponding lagged first-differences dated (t - 1) plus year dummies in the levels equations.

Table 5. Synthesis of input elasticities under different specifications.

		Robust Regressions	System-GMM	
			Predetermined explanatory variables	Endogenous explanatory variables
Cobb-Douglas	ε_K	0.46	0.23	0.33
	ε_E	0.54	0.91	1.05
Translog	ε_K	0.23	0.49	0.61
	ε_E	0.70	0.61	0.51

Note: In the case of the Cobb-Douglas matching function, $\varepsilon_K = \beta_K$ and $\varepsilon_E = \beta_E$.