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PURE RENT TAXATION AND GROWTH IN A TWO-SECTOR OPEN ECONOMY

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Pure Rent Taxation and Growth in a Two-Sector Open Economy*

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Abstract

In this paper, the incidence of a tax on pure rent is analyzed in an OLG two-sector small open economy, in which one sector produces a capital good and one sector a consumer good. Contrary to what is obtained by Feldstein (1977) in a one-sector closed economy, a rent tax does not necessarily foster wealth accumulation and economic growth. The accommodating scheme for the government budget plays a crucial role for the effects of pure rent taxation. A rent tax stimulates nonhuman wealth if distortionary taxes on wealth or on income from nonland inputs are alleviated. The mechanism spurring capital formation is brought into action, instead, only when the rent tax is matched by a fall in capital taxation or, if the capital sector is capital intensive, by an increase in government spending on the capital good.

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1 Introduction

In intertemporal optimizing models of capital accumulation with a non-reproducible asset, like land, the incidence of a tax on pure rent (i.e., a tax on income from the fixed asset) strictly depends on the demographic structure of the economy. When the economy is peopled by infinitely lived agents or by overlapping-generations with intergenerational altruism *à la* Barro (1974), a pure rent tax is never shifted. The reduction of the fixed asset price by the capitalized value of the tax is the only effect registered.

This result, realized *ante litteram* by Ricardo (1817) in his dynamic theory of rent, has been demonstrated by Calvo, Kotlikoff and Rodriguez (1979) (henceforth CKR) in a two-period OLG model with unimproved land, in which the utility of future generations enters the utility of current generations as in Barro (1974). Since the taxed activity cannot be modified, as the stock of land is inelastic, a pure rent tax, whose proceeds are either lump-sum transferred to consumers or used to increase unproductive government spending, is unshifted as it does not change the agents' well-being or the supply of capital and labor.[1]

In an overlapping-generations model of economic growth without operative intergenerational gifts and bequests, instead, a tax on land rent is partly shifted.[2] This result, discovered by Feldstein (1977), is due to the fact that the rise in the land rent tax, by initially reducing the value of land, increases saving available for investment in physical capital and hence stimulates economic growth through capital formation. The tax shifting stems from the fact that, as the capital stock accumulates, the return on land (like the wage

[1] By showing the strict connection between the Ricardian demographics (in which government debt is equivalent to lump-sum taxation when government spending is kept constant) and the Ricardian neutrality of a pure rent tax (that is, the fact that a tax on a fixed asset is inconsequential for the resource allocation), CKR curiously demonstrate the internal consistency and the epistemological solidness of Ricardo's (1817) theory.

[2] This would also be valid if a demographic setup *à la* Weil (1989) —that is, a model peopled by infinitely lived agents that are not altruistically linked and continuously enter the economy— were considered.

rate) rises, while the return on capital falls. Such changes in factor rewards may in the end paradoxically imply a possible increase in the land value.

It is the asset-valuation effect, induced by the change in the price of the fixed asset, that leads, through capital (the alternative asset to the taxed one) deepening, to a higher land rent to capital return ratio, hence causing the shifting of the tax.[3] The Feldstein (1977) result, obtained for a closed economy, is very robust, being independent of alternative uses of tax revenue.

Other contributions —like, for example, Fane (1984), Chamley and Wright (1987), Kotlikoff and Summers (1987), Buiter (1988), Eaton (1988), Ihori (1990), Deaton and Laroque (2001), and Petrucci (2006)— study the implications of pure rent taxation for capital accumulation and economic growth in life-cycle models with no intergenerational altruism.[4] Two aspects are particularly emphasized by this literature: the equivalence of land taxation with government debt manipulations in an overlapping-generations setting —discussed by Fane (1984) and Buiter (1988), but contained *in nuce* in CKR— and the effects of a rent tax shift in a small open economy facing perfect capital mobility —analyzed by Eaton (1988) and Petrucci (2006).

In an OLG economy without bequests, a land tax that is accompanied by the issuance of government debt (with the payments of principal and in-

[3] As observed by Chamley and Wright (1987), and Eaton (1988), there is a strict analogy between the 'Feldstein effect' —i.e., the capital spurring and growth enhancing effect of rent taxation— and the 'Tobin effect' — i.e., the positive effect of long-run inflation on capital accumulation and growth, discovered by Tobin (1965). In the Tobin (1965) analysis, capital represents the alternative asset to money, which is taxed through inflation. Such an analogy regards not only the mechanism underpinning the effects of (land rent and money) taxation on capital formation, but also the role of demographics. In fact, regarding the demographic aspect, money is superneutral when agents are infinite-lived (as shown by Sidrauski, 1967), while the 'Tobin effect' holds in an OLG model (as obtained by Drazen, 1981).

[4] Although the focus of Deaton and Laroque (2001) is not exactly the incidence of a pure rent tax, they employ a mechanism *à la* Feldstein (1977) to study the effects of land for housing on the growth process; they show that land markets, far from generating saving and growth, are inimical to capital formation.

terest on the newly issued bonds financed by land tax revenue) —i.e., an experiment in which the land tax is fully compensated— is neutral for the resource allocation. This equivalence of government debt and pure rent taxation —expression of the strict connection between the two aspects of Ricardo's (1817) theory underlined above— represents an extension of the cases in which government debt is not 'net wealth', surprisingly obtained when agent time horizons are finite and there are no bequests.

In a small open economy with an unrestricted access to the world capital market and an inelastic labor supply, a land rent tax does not affect economic growth, since the foreign interest rate, which is exogenous, pins down the capital stock (Eaton, 1988). By decreasing foreign investment, such a tax shift, however, raises welfare in the long-run. As shown by Petrucci (2006), when endogenous labor-leisure choices are considered in such an economic environment, a rent tax, instead, by stimulating nonhuman wealth and consumption, leads to a decline in labor hours and to a decumulation of capital if land tax revenue is lump-sum given to consumers. No effect on the capital stock is obtained, when land taxation is matched by an increase in government spending, while capital is ambiguously affected, if the offset is a cut in labor taxation.

All the articles on the macroeconomic effects of a rent tax, cited earlier, consider economies with one-sector of production. No scientific contribution has tried to understand or has explicitly taken into consideration so far the role of heterogeneity on the supply-side for the consequences of land taxation in models with finitely lived households. This lack of investigation is surprising, as the analysis of tax incidence in models with different sectors of production has a long tradition in public economics —starting from the Harberger (1962) seminal contribution— and a suggestion for studying pure rent taxation in a two-sector economy has been put forward by Feldstein (1977, p. 357).

In this paper, the incidence of a tax on pure rent is examined in a small open economy model of fixed investment with endogenous labor-leisure choices, taking into account sectoral differences in production. The analysis considers an overlapping-generations economy *à la* Blanchard-Yaari, facing perfect international capital mobility and incorporating the capital and consumer goods' decomposition as in Uzawa (1961 and 1964).[5]

The role of alternative government budget regimes accompanying land taxation for the valuation effect, saving and economic growth is examined with special attention. In particular, a rather articulate set of experiments is studied, in which a land tax rise is accompanied either by lifting other taxes in a revenue-neutral form —like taxes on nonhuman wealth, labor and physical capital— or by increasing sectoral government expenditure on consumer and capital goods.

Contrary to what is discovered by Feldstein (1977) in a one-sector closed economy, the general finding of the paper is that a rise in the pure rent tax does not necessarily foster nonhuman wealth and capital formation. The effects of rent taxation on wealth and economic growth critically depend on the accommodating scheme for the government budget.

In this paper it is shown that nonhuman wealth is stimulated and the capital stock is reduced in the long-run along with hours worked, when a wealth tax is exchanged for a land tax. If the compensatory finance implies either a cut in payroll taxation or an increase in government spending on the consumer good, a pure rent tax, instead, will leave aggregate capital and labor invariant. Nonhuman wealth will be increased by a fall in the labor tax, while it will remain unchanged when government spending is raised.

It is only when a tax on land rent is matched by a fall in capital taxation or, if the capital sector is relatively capital intensive, by a rise in government spending on the capital good that the mechanism spurring capital formation is brought into action. In these cases, steady state aggregate manhours remain unchanged. The compensatory increase in government spending on the

[5] Eaton (1987 and 1988), and Engel and Kletzer (1990) develop dynamic two-sector models with either financial autarky or perfect financial capital mobility, incorporating land and capital that are simultaneously used as specific factors of production and stores of value.

capital good leaves nonhuman wealth invariant.

The surprising effects of a tax on land rent on capital formation depend upon the fact that the capital stock, as the investment good is nontradable, is basically determined by the equilibrium condition on the capital good market. Depending on the government budget accommodating regime, such a condition should be considered in association with either the government budget constraint or the Blanchard-Yaari equilibrium condition for labor hours worked.

In the analysis of the paper, the endogeneity of the labor supply matters for the results only in the experiment based on the wealth tax compensatory finance, while the sectoral structure of the economy plays a crucial role for aggregate capital and economic growth only when government spending on the capital good is compensatorily accommodated.

The two-sector analysis of pure rent taxation is structured as follows. Section 2 presents the analytical setup, section 3 investigates the implications of land tax shifts under different budgetary schemes, and section 4 concludes.

2 The model

Consider a small open economy, facing a perfect international capital market and producing competitively two goods: a consumer good, Z_C, and a capital (investment) good, Z_I. The consumer good is supposed to be internationally tradable and perfectly substitutable with the foreign-produced consumer good, while the capital good is assumed to be internationally nontradable.[6]

The consumer good, considered as the numeraire, is produced, using capital, labor and land, by means of the following constant-returns-to-scale production function[7]

[6] The same trade structure of an open economy with capital and consumer industries *à la* Uzawa (1961 and 1964) can be found in Phelps (1988), and Phelps (1994, Ch. 13).

[7] The hypothesis that land, whose stock is inelastic, is specific to the C-sector does not affect the results obtained here.

$$Z_C = \Phi(K_C, L_C) + \omega T = L_C \phi(k_C) + \omega T, \tag{1}$$

where K_C and L_C are the capital stock and labor employed in the C-sector, respectively, T represents the stock of land, $\omega > 0$ a fixed technical coefficient (i.e., the constant marginal productivity of land), $\phi(k_C) \equiv \dfrac{\Phi(K_C, L_C)}{L_C} \equiv \Phi(\dfrac{K_C}{L_C}, 1)$, $k_C \equiv \dfrac{K_C}{L_C}$, $\phi' > 0$ and $\phi'' < 0$. It is assumed for simplicity that the production function is strongly separable in capital and labor, on the one side, and land, on the other.[8]

The production of the capital good employs capital and labor alone, according to the linearly homogeneous production function

$$Z_I = \Psi(K_I, L_I) = L_I \psi(k_I), \tag{2}$$

where K_I and L_I are sectoral capital and labor, respectively, $\psi(k_I) \equiv \dfrac{\Psi(K_I, L_I)}{L_I} \equiv \Psi(\dfrac{K_I}{L_I}, 1)$, $k_I \equiv \dfrac{K_I}{L_I}$, $\psi' > 0$, and $\psi'' < 0$.

As both labor and capital can be shifted instantaneously and costlessly between sectors, the marginal products of capital and labor are equalized across sectors; therefore, the input demand system can be expressed as

$$\phi'(k_C) = q\psi'(k_I) = R_K^f, \tag{3a}$$

$$\phi(k_C) - k_C \phi'(k_C) = q[\psi(k_I) - k_I \psi'(k_I)] = v^f, \tag{3b}$$

$$\omega = R_T^f, \tag{3c}$$

[8]The production function (1) is more general than it may appear at first sight. In fact, the qualitative results of the paper would remain unaffected if the production function $Z_C = \Gamma[\min(K_C, \kappa_C L_C), T]$ (where κ_C is a technical coefficient representing the fixed capital-labor ratio in the C-sector) were employed. If, instead, a generic production function $Z_C = \Lambda(K_C, L_C, T)$ were used, only some qualitative results concerning the sectoral allocation of inputs could be altered.

where q is the real price of the capital good (expressed in terms of the numeraire), R_K^f the cost of capital for firms, v^f the firm labor cost, and R_T^f the firm cost of land.

Factors of production are subject to taxation. The fiscal treatment of capital and labor is sector independent. The factor costs faced by firms are:

$$R_K^f \equiv (1+\tau_K)R_K^h, \tag{4a}$$

$$v^f \equiv (1+\tau_L)v^h, \tag{4b}$$

$$R_T^f \equiv (1+\tau_T)R_T^h, \tag{4c}$$

where τ_i for $i = K, L, T$ denotes an *ad valorem* tax rate on the i-th factor, R_K^h the real reward on capital received by households, v^h the household hourly real wage, and R_T^h real land rent obtained by households.

The behavior of consumers derives from the Yaari (1965) and Blanchard (1985) overlapping-generations approach with endogenous labor-leisure choices, as in Phelps (1994, ch. 16). Such an apparatus describes a continuous life-cycle framework (known as the 'perpetual youth' model) with no intergenerational altruism and bequests, in which the individuals' lifetime is uncertain as agents face a constant mortality rate θ for their entire life. New cohorts of equal size are born continuously. By postulating that the birth rate coincides with the death rate, the population, composed of cohorts of different age, remains constant and hence can be normalized to one.

Assuming logarithmic preferences, the demand-side of the model is given in aggregate terms by[9]

$$C = (\theta+\rho)(H+W), \tag{5a}$$

$$\widetilde{L} - L = \frac{C}{v^h}, \tag{5b}$$

[9]The detailed derivation of equations (5) is provided in Appendix 1.

$$\dot{H} = (r^h + \theta)H - v^h L, \tag{5c}$$

$$\dot{W} = r^h W + v^h L - C, \tag{5d}$$

where H is the stock of human wealth, W the stock of nonhuman wealth, r^h the interest rate faced by households, ρ the exogenous rate of time discount, and $\widetilde{L}$ the fixed time endowment.

In the Blanchard-Yaari apparatus, the aggregate Euler equation for consumption dynamics —obtained by combining (5a), (5c) and (5d)— is

$$\dot{C} = (r^h - \rho)C - \theta(\theta + \rho)W. \tag{5a'}$$

Because of perfect capital mobility, the domestic interest rate is given by the exogenous world interest rate r^*. Since we postulate that income from financial wealth is taxed domestically at the proportional rate τ_W, the interest rate that households face is given by $r^h = (1 - \tau_W)r^*$.

The asset menu is composed of physical capital, land and net foreign interest-earning assets F. Nonhuman wealth is given by the value of the capital stock measured in terms of the consumer good, qK, plus the value of land, pT (where p is the price of land expressed in terms of the numeraire), plus F; that is,

$$W = qK + pT + F. \tag{6}$$

F may be either positive or negative. The stock of nonhuman wealth is considered to be strictly positive; therefore, the condition $r^h > \rho$ must hold from (5a').

Assets are assumed to be perfectly substitutable. This requires the equalization of the expected returns to holding each asset:

$$r^* = \frac{R_K^h}{q} + \frac{\dot{q}}{q} - \delta, \tag{7a}$$

$$r^* = \frac{R_T^h}{p} + \frac{\dot{p}}{p}, \tag{7b}$$

where $\delta > 0$ represents the capital depreciation rate and perfect foresight has been assumed.

A tax on pure rent is a tax imposed on land rent perceived by households R_T^h. τ_T is qualitatively equivalent to such a tax. In fact, it can be easily shown that $\tau_T = \frac{\lambda}{(1-\lambda)}$ (with $\frac{d\tau_T}{d\lambda} > 0$), where λ denotes an *ad valorem* tax rate on R_T^h.[10]

The equilibrium of factor markets requires that inputs are fully employed at each point in time

$$K = K_C + K_I, \tag{8a}$$

$$L = L_C + L_I, \tag{8b}$$

$$T = 1, \tag{8c}$$

where the land endowment has been normalized to one, being exogenous.

The capital good, which is nontradable, is partly accumulated, partly used for replacing depreciated capital, and partly acquired by the government in amount G_I; that is,

$$Z_I = \dot{K} + \delta K + G_I. \tag{9}$$

As the consumption good is perfectly tradable internationally, the current account balance is

$$\dot{F} = Z_C - C - G_C + r^* F. \tag{10}$$

[10] Therefore, in the analysis of this paper, we will not consider a land tax rate λ explicitly as τ_T surrogates it.

According to (10), the sum of the trade balance —i.e., the consumer good output less private consumption and less government spending on the consumption good G_C— and interest income earned by holding net foreign assets gives the rate of accumulation of F.[11]

The government maintains its budget balanced by matching tax revenues with government expenditures continuously:

$$\tau_K R_K^h K + \tau_L v^h L + \tau_T R_T^h T + \tau_W r^* W = G_C + qG_I. \qquad (11)$$

The complete structural model in its short-run version is reported in Appendix 2.

The dynamic properties of the model are discussed in Appendix 3, where it is shown that the equilibrium is saddle-point stable.[12]

The effects of land rent taxation are studied by considering different alternative goverment budget regimes accompanying land taxation.[13] The two general schemes studied are: revenue-neutral changes in the tax mix, and compensatory increases in government spending.

In particular, within the scheme of *revenue-neutral changes in the tax mix*, the specific compensatory budgetary modes that will be alternatively considered are: 1.a) a reduction in τ_W; 1.b) a reduction in τ_L; 1.c) a reduction in τ_K.

When *compensatory changes of government spending* are taken into account, instead, we will specifically analyze: 2.a) a rise in G_C; 2.b) a rise in

[11] If foreign ownership of capital, K_f, and land, T_f, were allowed, like in Eaton (1988), the analysis would remain unchanged as the stock of net foreign assets F would be equal to the opposite of foreign investment B, defined as $B = K_f + qT_f$.

[12] The necessary condition that ensures saddle-point stability is $\theta(\theta + \rho) > r^h(r^h - \rho)$. It is assumed that such an inequality holds. Moreover, when $\bar{k}_I > \bar{k}_C$ (where overbarred variables denote steady state values), the sufficient condition $\psi + \delta(\bar{k}_C - \bar{k}_I) > (2r^* - \rho)(\bar{k}_C - \bar{k}_I)$ to have a saddle-point equilibrium must also be imposed; such an inequality satisfies the trace condition.

[13] The budgetary regime accompanying a change in the land tax has no qualitative implications for the properties of stability.

G_I.

3 Effects of land rent taxation

The analysis will only discuss the steady state effects of land taxation.

Because of the many experiments examined, the comparative statics consequences of pure rent tax shifts, under the diverse budgetary schemes, can be easily studied through some simplifying key-relationships, that will be derived in the following analysis.

Taking into account (3a), (4a) and (7a), solve for the capital intensity in the I-sector as follows

$$\bar{k}_I = k_I(\tau_K), \tag{12a}$$

where $k_I' = \dfrac{(r^* + \delta)}{\psi''} < 0$ and overbar denotes a long-run value.[14]

From (3a), (3b), (4a), (4b), and (12a), we obtain

$$\bar{k}_C = k_C(\tau_K), \tag{12b}$$

$$\bar{q} = q(\tau_K), \tag{12c}$$

where $k_C' = \dfrac{\psi \bar{q}^2 (r^* + \delta)}{\phi'' \phi} < 0$ and $q' = \dfrac{\bar{q}^2 (r^* + \delta)(\bar{k}_I - \bar{k}_C)}{\phi}$.

Factor rewards obtained by households can be expressed, once equations (3), (4) and (12) are employed, as

$$\bar{R}_K^h = \frac{R(\tau_K)}{(1 + \tau_K)}, \tag{13a}$$

$$\bar{v}^h = \frac{v(\tau_K)}{(1 + \tau_L)}, \tag{13b}$$

[14]Notice that τ_K is not overbarred since it is in general exogenous; only when the compensatory scheme involves the endogenous adjustment of capital taxation, will it be overbarred.

$$\bar{R}_T^h = \frac{\omega}{(1+\tau_T)}, \tag{13c}$$

where $R' = \dfrac{\psi\bar{q}^2(r^*+\delta)}{\phi} > 0$ and $v' = -\dfrac{\bar{k}_C\psi\bar{q}^2(r^*+\delta)}{\phi} < 0$.

By expressing (8a) as $\bar{K} = k_C(\tau_K)\bar{L}_C + k(\tau_K)\bar{L}_I$, once equations (12) have been employed, and using (8b), the following semi-reduced form for sectoral labor is obtained

$$\bar{L}_i = L_i(\bar{K}, \bar{L}, \tau_K), \tag{14}$$

where $L_{i,\bar{K}} = -\dfrac{1}{(\bar{k}_j - \bar{k}_i)}$, $L_{i,\bar{L}} = \dfrac{\bar{k}_j}{(\bar{k}_j - \bar{k}_i)}$, and $L_{i,\tau_K} = \dfrac{(k_i'\bar{L}_i + k_j'\bar{L}_j)}{(\bar{k}_j - \bar{k}_i)}$ for $i, j = C$, I and $i \neq j$.

By rewriting the adding-up constraint for labor (8b), after using (12a) and (12b), as $\bar{L} = \dfrac{\bar{K}_C}{k_C(\tau_K)} + \dfrac{\bar{K}_I}{k_I(\tau_K)}$, we obtain, once (8a) is taken into account, the following semi-reduced form for the sectoral capital stock

$$\bar{K}_i = K_i(\bar{K}, \bar{L}, \tau_K), \tag{15}$$

where $K_{i,\bar{K}} = -\dfrac{\bar{k}_i}{(\bar{k}_j - \bar{k}_i)}$, $\bar{K}_{i,\bar{L}} = \dfrac{\bar{k}_i\bar{k}_j}{(\bar{k}_j - \bar{k}_i)}$, and $\bar{K}_{i,\tau_K} = \dfrac{(k_i'\bar{k}_j\bar{L}_i + k_j'\bar{k}_i\bar{L}_j)}{(\bar{k}_j - \bar{k}_i)}$ for $i, j = C$, I and $i \neq j$.

The partial derivatives of equations (14) and (15) respond to the prescriptions of the Rybczynski theorem for sectoral labor and capital; differently from the standard Heckscher-Ohlin analysis in which the optimal capital-labor ratios of the two-sectors are constant at the optimal level, now the equilibrium capital intensities depend on the capital tax rate.

From (12a), (12b), and (14), after making use of (1) and (2), we get

$$\bar{Z}_C = Z_C(\bar{K}, \bar{L}, \tau_K), \tag{16a}$$

$$\bar{Z}_I = Z_I(\bar{K}, \bar{L}, \tau_K), \tag{16b}$$

where $Z_{C,\bar{K}} = -\dfrac{\phi}{(\bar{k}_I - \bar{k}_C)}$, $Z_{C,\bar{L}} = \dfrac{\phi k_I}{(\bar{k}_I - \bar{k}_C)}$, $Z_{C,\tau_K} = \dfrac{[k'_C \bar{L}_C(v^f + \phi' \bar{k}_I) + \phi \bar{L}_I k'_I]}{(\bar{k}_I - \bar{k}_C)}$, $Z_{I,\bar{K}} = \dfrac{\psi}{(\bar{k}_I - \bar{k}_C)}$, $Z_{I,\bar{L}} = -\dfrac{\psi \bar{k}_C}{(\bar{k}_I - \bar{k}_C)}$, and $Z_{I,\tau_K} = -\dfrac{[k'_I \bar{L}_I(v^f/\bar{q} + \psi' \bar{k}_C) + \psi \bar{L}_C k'_C]}{(\bar{k}_I - \bar{k}_C)}$.

Plugging Z_I from (16b) into the long-run version of (9), we get

$$Z_I(\bar{K}, \bar{L}, \tau_K) = \delta \bar{K} + G_I,$$

which can be solved for the capital stock as follows

$$\bar{K} = K(\bar{L}, \tau_K, G_I), \tag{17}$$

where $K_{\bar{L}} = \dfrac{\psi \bar{k}_C}{[\psi - \delta(\bar{k}_I - \bar{k}_C)]} > 0$, $K_{\tau_K} = \dfrac{[k'_I \bar{L}_I(v^f/\bar{q} + \psi' \bar{k}_C) + \psi \bar{L}_C k'_C]}{[\psi - \delta(\bar{k}_I - \bar{k}_C)]} < 0$, $K_{G_I} = \dfrac{(\bar{k}_I - \bar{k}_C)}{[\psi - \delta(\bar{k}_I - \bar{k}_C)]}$ and $\psi - \delta(\bar{k}_I - \bar{k}_C) = \dfrac{G_I}{\bar{L}_I} + \delta \bar{k}_C > 0$.

Equation (17) can be employed to derive the implications of τ_T for the aggregate capital stock. For this scope, the reduced-form of $\bar{K}$ is obtained from (17) when such an equation is used together with the solution for $\bar{L}$ and, if either τ_K or G_I are endogenously accommodated, the government budget constraint.

The determination of aggregate labor hours, which is independent of the sectoral structure of the economy, can be reconstructed as follows.[15]

Consider the sub-system composed of the long-run version of equations (5b), (5d) and (5a'). Define income from nonhuman wealth as $\bar{y}^W = (\bar{r}^h + \theta)\bar{W}$, where $\theta \bar{W}$ represents the actuarial dividend on nonhuman wealth. After using such a definition of $\bar{y}^W$, substituting out $\bar{C}$ through (5a') and rearranging, we have

[15] Such an analytical treatment follows from Hoon and Phelps (1996).

$$\frac{\bar{y}^W}{\bar{v}^h \widetilde{L}} = \frac{(r^h + \theta)}{r^h} \left(1 - \frac{2\bar{L}}{\widetilde{L}}\right), \tag{18a}$$

$$\frac{\bar{y}^W}{\bar{v}^h \widetilde{L}} = \frac{(r^h + \theta)(r^h - \rho)}{[\theta(\theta + \rho) - r^h(r^h - \rho)]} \frac{\bar{L}}{\widetilde{L}}, \tag{18b}$$

where $\theta(\theta + \rho) > r^h(r^h - \rho)$ as a necessary condition of saddle-point stability.[16] The income-from-wealth-to-wage ratio —i.e., $\frac{\bar{y}^W}{\bar{v}^h \widetilde{L}}$— is an important variable for the macroeconomic equilibrium in a Blanchard-Yaari model with endogenous labor-leisure choices.[17]

By solving equations (18) in terms of r^h, we obtain

$$\frac{\bar{y}^W}{\bar{v}^h \widetilde{L}} = \frac{(r^h + \theta)(r^h - \rho)}{\{2\theta(\theta + \rho) - r^h(r^h - \rho)\}} = y(r^h),\ y' > 0; \tag{19a}$$

$$\frac{\bar{L}}{\widetilde{L}} = \frac{\{\theta(\theta + \rho) - r^h(r^h - \rho)\}}{\{2\theta(\theta + \rho) - r^h(r^h - \rho)\}} = L(r^h),\ L' < 0. \tag{19b}$$

From (19), a pure rent tax shift is capable of changing the nonhuman-wealth-income-to-wage ratio and aggregate labor hours only when a compensatory reduction in the wealth tax rate τ_W occurs and therefore $r^h = (1 - \tau_W)r^*$ changes. In all the other financing experiments studied, shifts in τ_T do not modify $\frac{\bar{y}^W}{\bar{v}^h \widetilde{L}}$ and $\frac{\bar{L}}{\widetilde{L}}$.

Moreover, consumption can then be recovered residually from (5b), after the solution for $\bar{L}$ is considered, while the stock of nonhuman wealth is derived from (5a'), once the reduced-form for $\bar{C}$ is employed.

We are now ready to develop the comparative statics analysis of land taxation. The various experiments involving either modifications in the tax mix or changes in government spending are discussed in turn.

[16] See Appendix 3.

[17] See Phelps (1994), and Hoon and Phelps (1996).

3.1 Revenue-neutral rent tax shifts

In this sub-section, we assume, to simplify the analysis, that government spending on the capital good and tax rates that are not involved in each specific experiment are zero.

1.a) *Compensatory reduction in* $\bar{\tau}_W$

Suppose that an increase in the land tax is accompanied by an endogenous decline in the wealth tax rate that leaves fiscal revenue unchanged.[18]

The sectoral capital-labor ratios, $\bar{k}_C$ and $\bar{k}_I$, and the relative price of the capital good, $\bar{q}$, remain unaffected by (12). Nonland factor costs faced by firms are invariant. Also capital and labor rewards, described by (13), do not change.

The semi-reduced form for $\bar{\tau}_W$, which is endogenously adjusted (this is why such a variable is now overbarred), can be obtained from the government budget constraint. Assuming that $\tau_L = \tau_K = G_I = 0$ and using the definition of $\bar{y}^W$, equation (11) can be expressed as

$$\frac{\tau_T}{(1+\tau_T)}\omega + r^* \bar{v}^h \tilde{L} \frac{\bar{\tau}_W}{[(1-\bar{\tau}_W)r^* + \theta]} \frac{\bar{y}^W}{\bar{v}^h \tilde{L}} = G_C.$$

Solving this equation for $\bar{\tau}_W$, we obtain the following implicit function

$$\bar{\tau}_W = \tau_W\left(\frac{\bar{y}^W}{\bar{v}^h \tilde{L}}, \tau_T\right), \tag{20}$$

where $\tau_{W1} = -\frac{\bar{\tau}_W \bar{v}^h \tilde{L}}{(r^*+\theta)\bar{W}} < 0$ and $\tau_{W2} = -\frac{[(1-\bar{\tau}_W)r^*+\theta]}{(r^*+\theta)r^*\bar{W}} \frac{\omega}{(1+\tau_T)^2} < 0$.

The effects of the current tax experiment on labor and the nonwage-income-to-wage ratio are immediately obtained by operating through equations (19), once $\bar{r}^h$ is replaced by $(1-\bar{\tau}_W)r^*$. Substituting $\bar{\tau}_W$ from (20)

[18] In a small open economy, this experiment is qualitatively equivalent, in terms of resource allocation, to the case of a compensatory increase in welfare entitlements. Such an equivalence derives from the similar effects exerted by the tax shock on human wealth in the two budgetary regimes.

into the expression for $\bar{r}^h$ in (19a) and differentiating, we get the following multiplier for the income-from-wealth-to-wage ratio

$$\frac{d\left[\bar{y}^W/(\bar{v}^h\widetilde{L})\right]}{d\tau_T} = -\frac{r^* y' \tau_{W2}}{(1 + r^* y' \tau_{W1})} > 0, \tag{21a}$$

where $1 + r^* y' \tau_{W1} > 0$.[19] Plugging (20) into (19b), once the relationship $\bar{r}^h = (1 - \bar{\tau}_W) r^*$ is considered, totally differentiating and making use of (21a), we obtain

$$\frac{d\left(\bar{L}/\widetilde{L}\right)}{d\tau_T} = -\frac{r^* L' \tau_{W2}}{(1 + r^* y' \tau_{W1})} < 0. \tag{21b}$$

Therefore, the rise in τ_T , by increasing $\bar{r}^h$ through the fall in $\bar{\tau}_W$, leads to a higher nonwage-income-to-wage ratio and lower manhours worked.

The intuition of these effects is straightforward. The induced reduction in the wealth tax rate implies an intergenerational redistribution of resources from the older generations to the younger ones since the propensity to save of the young is higher than that of their elders. This increases aggregate saving, nonhuman wealth and the associated income. Consumption and hence leisure, as the wage rate faced by households is given, are consequently driven up. The rise in leisure implies a fall in labor hours.

From (17), the capital stock is reduced because of the drop in manhours.[20] Therefore, nonhuman wealth, given by $\bar{q}\bar{K} + \frac{\omega}{(1+\tau_T)r^*} + \bar{F}$, is pulled up by the accumulation of net foreign assets, which overwhelms the fall in the values of land and the capital stock.

Regarding the effects on sectoral production, the experiment is composite, in the sense that a simultaneous change in the aggregate supply of nonland

[19] Such an inequality is easily satisfied if $\bar{\tau}_W$ is not too high initially.

[20] As emphasized in the Introduction, this result sharply contrasts with Feldstein (1977) —who obtains that a reduction in the tax advantages to land, by depressing the land value, increases saving and capital— and Eaton (1988) —who finds that a land tax is neutral for the capital stock— but confirms what is found by Petrucci (2006) in a one-sector economy.

inputs is taking place. The reduction of aggregate capital and labor hours supplied imply in turn that a fall in capital and labor used in the two sectors occurs, irrespective of the relative factor abundance.[21]

A final consideration on the current experiment regards the case of an inelastic labor supply. In such a circumstance, a pure rent tax accompanied by a decline in wealth taxation increases nonhuman wealth and consumption, but exerts no consequences on the aggregate capital stock and the sectoral use of inputs. The reduction in the land value will be accompanied by a rise of net foreign assets of a greater size, which accounts for the increase in nonhuman wealth.

1.b) *Compensatory reduction in* $\bar{\tau}_L$

Consider the case in which the rise in τ_T is matched, in order to balance the government budget, by a fall in the labor tax rate.

As the real interest rate faced by households remains unaltered, $\frac{\bar{y}^W}{\bar{v}^h \tilde{L}}$ and $\frac{\bar{L}}{\tilde{L}}$ are unchanged. The aggregate capital stock, the sectoral capital intensities, capital and labor employed in both sectors and the relative price

[21] By using (14), (15) and (17), we get:

$$\frac{d\bar{L}_i}{d\tau_T} = \frac{1}{(\bar{k}_j - \bar{k}_i)[\psi - \delta(\bar{k}_I - \bar{k}_C)]} \left[\psi(\bar{k}_j - \bar{k}_C) - \delta\bar{k}_j(\bar{k}_I - \bar{k}_C)\right] \frac{\tilde{L}d\left(\bar{L}/\tilde{L}\right)}{d\tau_T},$$

and

$$\frac{d\bar{K}_i}{d\tau_T} = \frac{\bar{k}_i}{(\bar{k}_j - \bar{k}_i)[\psi - \delta(\bar{k}_I - \bar{k}_C)]} \left[\psi(\bar{k}_j - \bar{k}_C) - \delta\bar{k}_j(\bar{k}_I - \bar{k}_C)\right] \frac{\tilde{L}d\left(\bar{L}/\tilde{L}\right)}{d\tau_T}.$$

From these equations, we have that $\frac{d\bar{L}_C}{d\tau_T} < 0$, $\frac{d\bar{L}_I}{d\tau_T} < 0$, $\frac{d\bar{K}_C}{d\tau_T} < 0$, and $\frac{d\bar{K}_I}{d\tau_T} < 0$, as $\frac{d\left(\bar{L}/\tilde{L}\right)}{d\tau_T} < 0$ from (21b).

of the capital good remain unperturbed; also factor prices paid by firms are unchanged. Since $\bar{v}^f \equiv (1+\bar{\tau}_L)\bar{v}^h$ is constant, the fall in $\bar{\tau}_L$ raises the household take-home wage $\bar{v}^h$. The stock of nonhuman wealth, income from wealth and consumption are all consequently pulled up, from (5a') and (5c), in equal proportion to $\bar{v}^h$. Nonhuman wealth is brought up by the increase in $\bar{F}$.

1.c) *Compensatory reduction in* $\bar{\tau}_K$

Suppose now that the hike in τ_T leads to a reduction in $\bar{\tau}_K$.

The income-from-wealth-to-wage ratio and aggregate labor hours are not affected.

The cost of capital faced by firms is reduced, while the real reward on capital obtained by households may rise or fall depending on whether $\bar{k}_I$ is greater or lower than $\bar{k}_C$.[22] The firms' labor cost and the workers' take-home wage go up.

Therefore, as the household wage is pulled up, the effects of τ_T on financial wealth and consumption are positive, as $\frac{\bar{y}^W}{\bar{v}^h\tilde{L}}$ and $\frac{\bar{y}^W}{\bar{C}}$ are constant, while net foreign assets move ambiguously.

By assuming $\tau_L = \tau_W = G_I = 0$, the government budget constraint (11) can be solved for the capital tax rate as follows

$$\bar{\tau}_K = \tau_K(\bar{K}, \tau_T), \tag{22}$$

where $\tau_{K1} = -\frac{\bar{\tau}_K}{\bar{K}} < 0$ and $\tau_{K2} = -\frac{\omega}{(1+\tau_T)^2\bar{K}\bar{q}(r^*+\delta)} < 0$.

Plugging (22) into (17) for $\bar{\tau}_K$, and totally differentiating, we obtain

$$\frac{d\bar{K}}{d\tau_T} = \frac{K_{\tau_K}\tau_{K2}}{(1-K_{\tau_K}\tau_{K1})} > 0,$$

where $K_{\tau_K}\tau_{K1} < 1$ if $\bar{\tau}_K$ is not too high initially.

[22] In fact, it is not difficult to show that sgn $\left(\frac{d\bar{R}_K^h}{d\tau_T}\right)$=sgn $(\bar{k}_I - \bar{k}_C)$.

Thus, the aggregate capital stock is increased as the firm's cost of capital is decreased by the drop in the capital tax rate.

The fall in $\bar{R}_K^f$ drives up the capital-labor ratio in each sector. The relative price of the capital good moves ambiguously; it is pulled up (down) if the capital sector is relatively capital (labor) intensive.

Regarding the sectoral use of inputs, the capital stock used in each sector is increased. If $\bar{k}_I >(<) \bar{k}_C$, the fall in the capital tax rate lowers (raises) labor employed in the C-sector, while increases (shrinks) labor employed in the I-sector.

3.2 Rent tax shifts with increased government spending

2.b) *Compensatory increase in* $\bar{G}_C$

A rise in government spending on the consumption good that accompanies the increase in the pure rent tax is neutral for the macroeconomic equilibrium. Nothing changes except for a reduction in the land value and a concomitant increase in net foreign assets.[23]

Notice that a shift to increased pure rent taxation offset by a lighter value added tax, if it were considered, would occasion the same qualitative results observed in the current experiment.[24]

2.b) *Compensatory increase in* $\bar{G}_I$

[23] This is because the income-from-wealth-to-wage ratio and labor hours are left undisturbed by the rise in τ_T, as the interest rate faced by households is given. Nonhuman wealth and consumption are not affected as the workers' wage is unaltered. Nor does the capital stock change as aggregate manhours are given. Because of the reduction in the land price and the fact that the value of physical capital $\bar{q}\bar{K}$ is constant, the invariance of nonhuman wealth is guaranteed by an increase in the holdings of net foreign assets.

[24] This is not surprising as a shift in consumption taxation accompanied by a balanced-budget change in $\bar{G}_C$ does not affect the allocation of resources.

When the additional proceeds from a land tax rise are employed to increase government expenditure directed toward the capital good, $\frac{\bar{y}^W}{\bar{v}^h \tilde{L}}$ and $\frac{\bar{L}}{\underset{\sim}{L}}$, the capital-labor ratios in both sectors, nonland factors' rewards, and the relative price of the capital good do not change. Nonhuman wealth and consumption are invariant since the household wage does not move.

As sgn(K_{G_I})=sgn($\bar{k}_I - \bar{k}_C$) in equation (17), what happens to the aggregate capital stock depends upon whether the I-sector is relatively more or less capital-intensive than the C-sector. If it is, that is $\bar{k}_I > \bar{k}_C$, then $\bar{K}$ will increase; otherwise, if $\bar{k}_I < \bar{k}_C$, $\bar{K}$ will decline.

Capital and labor used in the C-sector fall, while capital and labor employed in the I-industry rise.[25] The antithetic behavior of $\bar{K}_C$ and $\bar{K}_I$ explains why the overall effect on $\bar{K}$ depends upon whether $\bar{k}_I$ is higher or lower than $\bar{k}_C$.

The net claims on foreigners are increased as $\bar{Z}_C + r^* \bar{F}$ remains constant, from (10), and $\bar{Z}_C$ falls.

[25] From (14), (15) and (17), we get:

$$\frac{d\bar{L}_i}{d\tau_T} = -\frac{(\bar{k}_I - \bar{k}_C)}{(\bar{k}_j - \bar{k}_i)[\psi - \delta(\bar{k}_I - \bar{k}_C)]}\frac{d\bar{G}_I}{d\tau_T},$$

and

$$\frac{d\bar{K}_i}{d\tau_T} = -\frac{\bar{k}_i(\bar{k}_I - \bar{k}_C)}{(\bar{k}_j - \bar{k}_i)[\psi - \delta(\bar{k}_I - \bar{k}_C)]}\frac{d\bar{G}_I}{d\tau_T},$$

where $\frac{d\bar{G}_I}{d\tau_T} = \frac{\bar{q}\omega}{(1+\tau_T)^2} > 0$ if $\tau_W = \tau_L = \tau_K = 0$.

Therefore, the sectoral effects of land taxation are: $\frac{d\bar{L}_C}{d\tau_T} < 0$, $\frac{d\bar{L}_I}{d\tau_T} > 0$, $\frac{d\bar{K}_C}{d\tau_T} < 0$, and $\frac{d\bar{K}_I}{d\tau_T} > 0$.

4 Concluding remarks

Concluding what is now a classical survey on tax incidence, Kotlikoff and Summers (1987) state that: "Economics is at its best when it offers important insights that contradict initial, casual impressions. The theory of tax incidence provides a rich assortment of such insights. ...... The study of tax incidence is both fun, because it offers such surprising findings, and very important, because of its implications about the impacts of government policies. ......".

Consistently with the above reflections by Kotlikoff and Summers (1987), this contribution has offered further confirmation of the heuristic richness of tax analysis. In fact, insights that contradict direct, superficial impressions regarding the incidence of a tax on pure rent in a two-sector small open economy have been provided, and unexpected results that depart from those of one-sector analyses have been obtained.

The implications of land rent taxation for wealth formation, the aggregate capital stock and economic growth in a life-cycle open economy, in which one sector produces a capital good and one sector a consumer good, have been the principle focus of the paper. In the setup employed, the consumer good is perfectly tradable at an international level, while the capital good can only be exchanged domestically.

Two general budgetary schemes accompanying the increase of land taxation have been studied: one in which the structure of distortionary taxation is modified in a revenue-neutral form, and one in which the rise in the tax revenue is balanced by the compensatory increase in government expenditure on sectoral goods. The compensatory scheme for the government budget is determinant for the incidence of rent taxation.[26]

We obtain that land taxation is not necessarily effective in terms of non-

[26] In the case studied by Feldstein (1977) of a one-sector closed economy with overlapping-generations, in which a positive effect of land taxation on capital formation, economic growth and wealth is obtained, the use of tax revenue is, instead, unimportant for the results.

human wealth formation, capital stimulation and economic growth enhancement in a two-sector small open economy facing perfect capital mobility. The increase in wealth, when obtained, does not necessarily spur capital accumulation, while the invariance of wealth may be associated with disparate effects on capital formation.

Instead of summarizing the results of the analysis according to the different budget accommodating experiments studied, as done in the paper, we synthesize them in terms of the implications on the main aggregate variables. From such a perspective, the findings of the paper are as follows:

i) *Nonhuman wealth and consumption*

Nonhuman wealth is raised by land taxation when taxes on wealth, labor or capital are compensatorily adjusted, while it is left unchanged when the government expenditure on sectoral goods is modified. Wealth is stimulated through the rise of the saving return if a wealth tax is decreased. In the case of labor or capital tax reduction, instead, the increase in wealth stems from the increase in the household take-home wage.

Consumption always moves in the same direction as the stock of nonhuman wealth.

ii) *Aggregate capital*

Pure rent tax shifts are an ineffective instrument to influence capital accumulation if payroll taxation or government spending on the consumer good are compensatorily adjusted. Land tax shifts are detrimental for capital formation and economic growth when wealth taxation is decreased or, if the consumer good sector is relatively capital intensive, government spending on the capital good is increased. The 'Feldstein effect' on the capital stock is obtained —that is, capital accumulation is spurred— if capital taxes are reduced or, when the capital sector is capital intensive, government spending on the investment good is raised.

iii) *Aggregate labor hours*

Since in the Blanchard-Yaari setup with endogenous labor-leisure choices, the ratio of income from wealth to the worker take-home wage is the ultimate determinant of labor hours supplied, the nonneutrality of a pure rent tax for hours worked is obtained when nonwage income and the household wage change by different proportions. This happens only when the compensatory finance involves an adjustment in the tax rate on nonhuman wealth.

iv) *Nonland factor prices and rewards*

The firm's cost of capital is always invariant except for the case in which the accommodating finance leads to a fall in capital taxation; in such a case it falls. The real reward on capital perceived by households moves ambiguously when the proceeds from land taxation are employed to cut capital taxes, but remains unchanged in all the other experiments.

The cost of labor for firms is only modified by the pure rent tax when a concomitant cut in capital taxation takes place, in which case it is pulled up. The take-home wage, instead, is increased only if the offset is a fall in labor or capital taxation.

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Pure Rent Taxation and Growth
in a Two-Sector Open Economy

MATHEMATICAL APPENDICES

APPENDIX 1
Microeconomics of the demand–side

This appendix provides the derivation of the aggregate behavior of consumers, namely equations (5) of the text.

Assuming that the individual utility is logarithmic in consumption, c, and leisure, $\widetilde{l} - l$ (where $\widetilde{l}$ is time endowment and l represents labor hours supplied), at each instant t a consumer born at time $s \leq t$ solves the following problem

$$\max \int_t^{\infty} \left\{ \ln c(s,j) + \ln \left[\widetilde{l} - l(s,j) \right] \right\} \exp[-(\theta + \rho)(j - t)] dj \qquad (1.1)$$

subject to the intertemporal budget constraint

$$\int_t^{\infty} c(s,j) \exp\{-\int_t^{n} [r^h(t) + \theta](j - t) dn\} dj = w(s,t) + h(s,t) \qquad (1.2)$$

where $h(s,t)$ is the consumer's human wealth, given by

$$h(s,t) = \int_t^{\infty} v^h(j) l(s,j) \exp\{-\int_t^{n} [r^h(t) + \theta](j - t) dn\} dj,$$

and $w(s,t)$ denotes nonhuman wealth of a consumer born at time s; θ gives the mortality rate (exogenous), ρ the rate of time preference (exogenous),

$r^h(t)$ the real interest rate faced by consumers, and $v^h(t)$ the hourly real wage.[27]

The optimality conditions for the individual problem (1.1)-(1.2) are

$$c(s,t) = (\theta+\rho)[w(s,t) + h(s,t)],$$

$$\widetilde{l} - l(s,t) = \frac{c(s,t)}{v^h(t)},$$

$$\frac{d}{dt}c(s,t) = [r^h(t) - \rho]x(s,t).$$

Aggregating over all the cohorts and omitting the time index, the demand-side of the model can be expressed as

$$C = (\theta+\rho)(W+H), \tag{1.3a}$$

$$\widetilde{L} - L = \frac{C}{v^h}, \tag{1.3b}$$

$$\dot{H} = (r^h + \theta)H - v^h L, \tag{1.3c}$$

$$C + \dot{W} = r^h W + v^h L, \tag{1.3d}$$

[27] Note that the intertemporal budget constraint (1.2) is obtained by integrating the flow budget constraint forward

$$\frac{d}{dt}w(s,t) = [r^h(t)+\theta]w(s,t) + v^h(t)l(s,t) - c(s,t),$$

and using the solvency condition precluding Ponzi schemes

$$\lim_{j\to\infty} w(j,t)\exp\{-\int_t^n [r^h(t)+\theta](j-t)dn\} = 0.$$

where capital letters denote aggregate variables of the corresponding individual variables.[28] Equations (1.3) correspond to equations (5) of the main text.

By using equations (1.3), the Blanchard-Yaari equation for consumption growth can be easily obtained:

$$\dot{C} = (r^h - \rho)C - \theta(\theta + \rho)W. \tag{1.3a'}$$

This is equation (5a').

[28] A generic aggregate variable is defined as $X(t) = \int_{-\infty}^{t} x(s,t)\theta \exp[\theta(s-t)]ds$, where $x(s,t)$ indicates the corresponding individual variable.

APPENDIX 2
The structural model

The short-run version of the structural model is

$$\phi'(k_C) = q\psi'(k_I), \tag{2.1}$$

$$\phi(k_C) - k_C\phi'(k_C) = q[\psi(k_I) - k_I\psi'(k_I)], \tag{2.2}$$

$$\phi(k_C) - k_C\phi'(k_C) = (1 + \tau_L)v^h, \tag{2.3}$$

$$K = K_C + K_I, \tag{2.4}$$

$$L = L_C + L_I, \tag{2.5}$$

$$\widetilde{L} - L = \frac{C}{v^h}, \tag{2.6}$$

$$\frac{\tau_K\phi'(k_C)K}{(1+\tau_K)} + \tau_L v^h L + \frac{\tau_T\omega}{(1+\tau_T)} + \tau_W r^*(qK + p + F) = G_C + qG_I, \tag{2.7}$$

$$\dot{C} = [(1 - \tau_W)r^* - \rho]C - \theta(\theta + \rho)(qK + F + p), \tag{2.8}$$

$$\dot{q} = (r^* + \delta)q - \frac{\phi'(k_C)}{(1+\tau_K)}, \tag{2.9}$$

$$\dot{p} = r^*p - \frac{\omega}{(1+\tau_T)}, \tag{2.10}$$

$$\dot{K} = L_I\psi(k_I) - \delta K - G_I, \tag{2.11}$$

$$\dot{F} = \phi(k_C)L_C - C - G_C + r^*F. \tag{2.12}$$

and $k_C \equiv \dfrac{K_C}{L_C}$ as well as $k_I \equiv \dfrac{K_I}{L_I}$.

The endogenous variables are k_C, k_I, L_C, L_I, K_C, K_I, q, p, K, L, C, F, v^h and one fiscal endogenous variable, according to the compensatory financing regime, obtained from the government budget constraint (2.7).

APPENDIX 3
Analysis of stability

The analysis of stability is discussed when the compensatory finance is based on the endogenous accommodation of G_C.[29] Assume, for the sake of simplicity, that $\tau_K = \tau_L = \tau_W = G_I = 0$.

By considering equations (2.1)-(2.3), we obtain

$$k_C = k_C(q), \tag{3.1a}$$

$$k_I = k_I(q), \tag{3.1b}$$

$$v^h = v^h(q), \tag{3.1c}$$

where $k_C^{'} = \dfrac{\psi}{\phi''(\bar{k}_I - \bar{k}_C)}$, $k_I^{'} = \dfrac{\phi}{\bar{q}^2\psi''(\bar{k}_I - \bar{k}_C)}$, and $v^{h\prime} = -\dfrac{\bar{k}_C\psi}{(\bar{k}_I - \bar{k}_C)}$.

Using (2.6) together with (3.1c), yields

$$L = L(C, q), \tag{3.1d}$$

where $L_C = -\dfrac{1}{\bar{v}^h} < 0$ and $L_q = -\dfrac{\bar{k}_C\psi\bar{C}}{\bar{v}^{h2}(\bar{k}_I - \bar{k}_C)}$.

From (2.4) and (2.5), after using (3.1a) and (3.1b), we have

$$L_C = L_C(C, K, q), \tag{3.1e}$$

$$L_I = L_I(C, K, q), \tag{3.1f}$$

where $L_{C,C} = -\dfrac{\bar{k}_I}{\bar{v}^h(\bar{k}_I - \bar{k}_C)}$, $L_{C,K} = -\dfrac{1}{(\bar{k}_I - \bar{k}_C)}$,

[29] The budgetary regime accompanying a change in the land tax has no qualitative implications for the dynamic properties of the model.

$$L_{C,q} = \frac{1}{(\bar{k}_C - \bar{k}_I)^2}\left[\left(\frac{\bar{L}_I\phi}{\bar{q}^2\psi''} + \frac{\bar{L}_C\psi}{\phi''}\right) - \frac{\bar{C}\bar{k}_C\bar{k}_I\psi}{\bar{v}^{h2}}\right],\ L_{I,C} = \frac{k_C}{\bar{v}^h(\bar{k}_I - \bar{k}_C)},$$

$$L_{I,K} = \frac{1}{(\bar{k}_I - \bar{k}_C)},\text{ and } L_{I,q} = -\frac{1}{(\bar{k}_C - \bar{k}_I)^2}\left[\left(\frac{\bar{L}_C\phi}{\bar{q}^2\psi''} + \frac{\bar{L}_I\psi}{\phi''}\right) - \frac{\bar{C}\bar{k}_C^2\psi}{\bar{v}^{h2}}\right].$$

Let us consider the dynamic equations. Equation (2.10) —that is, $\dot{p} = r^*p - \dfrac{\omega}{(1+\tau_T)}$— is an unstable differential equation in p. A finite long-run value for p is obtained if and only if $r^*p = \frac{\omega}{(1+\tau_T)}$; such a condition implies that the price of land is constant over time and jumps from one equilibrium to another in response to exogenous τ_T shifts.

The dynamics require the analysis of a four equation system —given by (2.8), (2.9), (2.11) and (2.12)— with two jump variables, q and C, and two predetermined variables, K and F.

Using (2.8), (2.9), (2.11) and (2.12) together with (3.1a), (3.1b), (3.1e) and (3.1f), and taking into account the fact that p is a jump variable, as discussed earlier, the model can be reduced to the following system of differential equations linearized around the steady state (written in a compact matrix form)

$$\begin{bmatrix}\dot{q}\\ \dot{C}\\ \dot{K}\\ \dot{F}\end{bmatrix} = \begin{bmatrix} -\dfrac{\phi}{\bar{q}(\bar{k}_I - \bar{k}_C)} & 0 & 0 & 0\\ -\theta(\theta+\rho)\bar{K} & (r^*-\rho) & -\theta(\theta+\rho)\bar{q} & -\theta(\theta+\rho)\\ 0 & \psi L_{I,C} & (\psi L_{I,K} - \delta) & 0\\ 0 & \phi L_{C,C} - 1 & \phi L_{C,K} & r^* \end{bmatrix}\begin{bmatrix} q-\bar{q}\\ C-\bar{C}\\ K-\bar{K}\\ F-\bar{F}\end{bmatrix} \tag{3.3}$$

where $\dfrac{\partial\dot{q}}{\partial(q-\bar{q})} = (r^*+\delta) - \phi'' k_C' = -\dfrac{\phi}{\bar{q}(\bar{k}_I - \bar{k}_C)}$.

The transition matrix in (3.3) must have two eigenvalues with positive real parts, associated with the forward-looking variables q and C, and two eigenvalues with negative real parts, associated with the predetermined variables K and F.

The determinant and the trace of the Jacobian in (3.3) are

$$| J |= \frac{\phi \left\{ [\theta(\theta+\rho) - r^*(r^*-\rho)][\psi - \delta(k_I - k_C)]\bar{v}^h + \theta(\theta+\rho)[\psi(\bar{v}^h + \bar{q}\delta k_C) - \phi\delta\bar{k}_I] \right\}}{\bar{q}(\bar{k}_I - \bar{k}_C)^2\bar{v}^h};$$

$$\text{tr}(J) = -\frac{\phi}{\bar{q}(\bar{k}_I - \bar{k}_C)} + 2r^* - \rho + \frac{\psi}{(\bar{k}_I - \bar{k}_C)} - \delta.$$

The determinant of the Jacobian has to be positive, as required by the nature of the dynamic variables q, C, K and F. This requires that the necessary condition $\theta(\theta+\rho) > r^*(r^*-\rho)$ must be satisfied, being plausibly $\psi(\bar{v}^h + \bar{q}\delta k_C) > \phi\delta\bar{k}_I$.[30]

Denoting a generic eigenvalue of the state matrix by η_i (for $i = 1$, 2, 3, and 4), the expression of the eigenvalue associated with (2.9) is given by $\eta_1 = -\frac{\phi}{\bar{q}(\bar{k}_I - \bar{k}_C)}$, which can be either positive or negative depending on the relative capital intensities. This implies that the dynamic behavior of the economy depends upon the sectoral capital-labor ratios; therefore, the two cases $\bar{k}_I > \bar{k}_C$ and $\bar{k}_I < \bar{k}_C$ need to be discussed separately.

Case $\bar{k}_I > \bar{k}_C$. When the capital good sector is more capital intensive, then $\eta_1 < 0$. Hence, since $| J |> 0$ and $\text{tr}(J) - \eta_1 = \eta_2 + \eta_3 + \eta_4 > 0$,[31] there are necessarily two roots with a positive real part and two roots with a negative real part, thus ensuring that the equilibrium is saddle-point stable.

Case $\bar{k}_C > \bar{k}_I$. If the capital intensity of the consumption good sector exceeds that of the capital good sector, then $\eta_1 > 0$. Now a sufficient condition for saddle-point stability is that $\text{tr}(J) - \eta_1 = \eta_2 + \eta_3 + \eta_4$ is negative. This is ensured by the inequality $\psi + \delta(\bar{k}_C - \bar{k}_I) > (2r^* - \rho)(\bar{k}_C - \bar{k}_I)$. It is assumed that such an inequality holds.

Note that in this case, equation (2.9), once (3.1a) is employed, is an unstable differential equation in q. A finite long-run value for q is obtained if and only if $(r^* + \delta)q = \phi'[k_C(\bar{q})]$; such a condition implies that the relative price of the capital good remains constant over time, eventually jumping from

[30]Note that the condition $\theta(\theta+\rho) > r^*(r^*-\rho)$ also ensures saddle-point stability in a one-sector economy.

[31]This is because $r^* > \rho$ and $\psi - \delta(k_I - k_C) = \frac{G_I}{L_I} + \delta k_C > 0$.

one equilibrium to another in response to exogenous τ_T shocks that change its long-run value. Therefore, in this case, also k_C, k_I, and v^h exhibit jump solutions, with no transitional dynamics.