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DEVIATING FROM THE FRIEDMAN RULE: A GOOD IDEA WITH ILLEGAL IMMIGRATION?

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Deviating from the Friedman Rule: A Good Idea with Illegal Immigration?

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Abstract

This paper studies the optimal inflation rate in a transactions costs model with illegal immigration. Although unauthorized immigrants use domestic money for making transactions and consume in the host country, their welfare does not enter the objective function of the Ramsey planner, because of their unofficial status. In this environment, the Friedman rule is nonoptimal, when only an income tax is available, as the inflation tax makes it possible to collect revenues from illegal immigrants, who are difficult to subject to taxation. When a consumption tax —that illegal immigrants have to pay when buying consumption goods in the host country— is also available, the zero inflation tax prescription is efficient only if the consumption-money ratio of domestic consumers is not greater than the illegal immigrants' one.

JEL classification: E31, E52, E61, E63, O41.

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1 Introduction

Avoiding taxing real money balances —i.e., setting the opportunity cost of holding money to zero— is optimal in many standard intertemporal optimizing monetary models with infinite-lived consumers. This policy prescription — known as the Friedman rule, since initially advocated by Friedman (1969) from a first-best perspective— maximizes the consumer surplus also when distortionary taxes, used to finance exogenous government spending, are available. The validity of the zero inflation tax result in a second-best setting with full price flexibility has received support, among others, from Kimbrough (1986), Chari, Christiano and Kehoe (1996), Correia and Teles (1996 and 1999), Chari and Kehoe (1999), and De Fiore and Teles (2003).[1]

The consideration of tax restrictions —associated with factors of productions, monopoly profits, pure rents and informal sectors— implies, instead, that setting a positive nominal interest rate is efficient from a second-best ground. These cases —that give support to the Phelps (1973) idea of optimally using the inflation tax along with all other available taxes to collect a given amount of revenue— are investigated, for example, by Faig (1988), Vegh (1989), Woodford (1990), Guidotti and Vegh (1993), Mulligan and Sala-i-Martin (1997), Nicolini (1998), Schmitt-Grohè and Uribe (2004 and 2010), Cunha (2008), and Petrucci (2011).[2]

Schmitt-Grohè and Uribe (2010 and 2012) have recently found another

[1] The second-best optimality of the Friedman rule is generally justified on the basis of the Diamond and Mirrlees (1971) public finance principle on production efficiency. According to such a normative principle, when the production function is constant returns to scale, intermediate goods should be exempted from taxation, as taxes are levied only on final goods. As money is an intermediate input in economies with a transaction technology, the optimal inflation tax has to be zero to preserve production efficiency. See, for example, Christiano, Chari and Kehoe (1996), Chari and Kehoe (1999), and Kocherlakota (2005).

Because of the functional equivalence between models with money in the utility function and models with liquidity costs, demonstrated by Feenstra (1986), the way in which money is introduced into the economy is basically irrelevant for the optimality of the Friedman rule with distortionary taxation. See Chari and Kehoe (1999).

[2] From a public finance perspective, when there are tax limitations, deviating from the Friedman rule is optimal as the Diamond and Mirrless principle (1971) no longer holds; see Munk (1980), and Chari and Kehoe (1999).

reason that justifies the optimality of collecting revenues from seignorage: the presence of a foreign demand for domestic currency.[3] In such a circumstance, agents that hold domestic currency (and hence are hit by the inflation tax) are nationals, whose welfare enters the planner's objective function, and foreigners, whose welfare does not matter for the social planner. In this environment, extracting resources from the rest of the world that hold domestic currency by means of the inflation tax is optimal, because it allows the government, benevolent toward domestic consumers, to increase the amount of resources available for nationals' consumption.[4]

In this paper, we study the issue of optimal monetary policy in an intertemporal optimizing transactions costs model with illegal immigration when prices are fully flexible. The analytical setup employed here to study the optimal inflation rate is quite similar, although based on different macroeconomic underpinnings, to the Schmitt-Grohè and Uribe (2010 and 2012) one that considers the presence of a foreign demand for domestic currency. In fact, analogously, the social welfare function coincides with the domestic consumers' utility function as a benevolent Ramsey planner does not take into account the welfare of illegal immigrants (despite the fact that they demand domestic money and consume in the host country), because of their unofficial status. Moreover, as in the Schmitt-Grohè and Uribe (2010 and 2012) contribution, where foreigners that hold domestic money cannot be taxed directly by the domestic fiscal authority, in a model with illegal immigration, unauthorized foreign workers are difficult to tax on an income basis in the host country.

In addition to the above analytical similarities with the Schmitt-Grohè and Uribe (2010 and 2012) investigation, the study of the optimal inflation tax with illegal immigration is interesting for the following reasons. First, the phenomenon of illegal immigration deeply affects the economic life of

[3] A case that is particularly relevant, from an empirical point of view, for the U.S. dollar and the euro.

[4] Notice that also in the Schmitt-Grohè and Uribe (2010 and 2012) contribution, such a deviation from the zero seignorage result derives, like in other second-best cases previously mentioned, from a sort of tax limitation, given by the impossibility for the domestic fiscal authority to collect resources from foreigners that hold domestic money through fiscal instruments other than the inflation tax.

many advanced and emerging countries.[5] Although different macroeconomic aspects of such a phenomenon have been largely analyzed,[6] the issue of optimal monetary policy in a world with illegal immigration has not been studied.

Second, as domestic output is absorbed by consumption costs (which include transaction costs) of unauthorized foreign workers, their money holding indirectly enters the feasibility constraint. Therefore, even though illegal immigrants' welfare does not enter the social welfare function, their money demand has to be taken into account by the Ramsey planner when setting the optimal inflation rate.

Third, illegal immigration involves *de facto* limitations on income taxation, thus implying incompleteness of the tax system. In fact, unauthorized aliens, who work illegally, are difficult to subject to income taxation (because they are hardly detected), while domestic residents are in general taxed without restrictions. As illegal immigrants hold domestic money and buy consumption goods in the host country, inflation and consumption taxes represent instruments that can be used by the social planner to tax them, thus replacing the missing ones. The implications of these fiscal instruments for the optimal monetary policy are epistemologically worth studying.

We discover that in an environment with illegal immigrants, setting a positive inflation tax may be optimal in a second-best sense, depending on whether the tax system is restricted or not, while the first-best monetary policy is the Friedman rule if all agents using domestic money are compensated for the inflation tax.[7]

Specifically, when only income taxation is available, it is optimal to deviate from the Friedman rule. This is because illegal immigrants, who cannot

[5] The European Union, India, and the U.S., among others, are characterized in a relevant way by illegal immigration. For the relevance of the phenomenon in the U.S., see, for example, Hofer, Rytina and Baker (2012) and the "Jobs, Workers and Skills" Chapter of the *2013 Economic Report of the President.*

[6] These are, for example, the implications for domestic residents' welfare, the domestic job displacement impact, the effects on national resource allocation, the consequences for capital accumulation and economic growth, etc. See, among others, Ethier (1986), Bond and Chen (1987), Djajic (1987), Hazari and Sgro (2003), Palivos (2009) and Liu (2010).

[7] In a world without distortionary taxes, the Friedman rule is not optimal if illegal immigrants are not compensated for the inflation tax and seignorage collected from them is lump-sum given to domestic consumers.

be taxed by using income taxation, may be taxed through inflation on their holding of domestic money.[8] Therefore, setting a positive nominal interest rate allows the fiscal authority to extract resources from those whose welfare is immaterial for the Ramsey planner.

When also consumption taxation is at the disposal of the social planner, the discrepancy between the consumption-money ratio of domestic consumers and the one of illegal workers is crucial for determining whether the optimal monetary policy should obey to the Friedman rule or not.

If the consumption-money ratio of illegal immigrants is equal to the corresponding ratio of domestic workers, the inflation tax should be zero. In this case, in terms of revenue collection, a consumption tax is equivalent to a tax on real money balances (that is, an implicit consumption tax). As the consumption tax burden is balanced between domestic consumers and illegal immigrants (because they have the same consumption-money ratio), the inflation tax represents a less efficient way to finance a given flow of government spending than an explicit consumption tax.

If instead the consumption-money ratio of domestic consumers is higher than the one of illegal workers,[9] it is Ramsey optimal to set a positive nominal interest rate. In this case, as consumption taxation hits relatively more nationals than undocumented foreign workers, the inflation tax is required to collect more resources from illegal immigrants, thus balancing the tax burden across different agents and increasing the resources available for consumption by those who matter for the social planner.

The paper is structured as follows. Section 2 builds an intertemporal optimizing monetary model with transactions costs and illegal immigration. Section 3 analyzes the optimal inflation tax when only income taxation is available. Section 4 investigates the implications of introducing a consumption tax on the efficient monetary policy. Section 5 concludes.

[8] A similar idea is provided by Nicolini (1998), Cavalcanti and Villamil (2003), and Schmitt-Grohè and Uribe (2010) for economies with informal sectors and no immigration.

[9] Such a case is plausible since unauthorized immigrants may use domestic money more intensively than domestic workers in order to be in hiding and to keep operating illegally.

2 The model

Consider a monetary economy, peopled by infinite-lived domestic consumers, who do not emigrate, and foreign workers, who enter and work in the domestic country illegally. Immigration is only of the illegal type. Illegal aliens are expatriated to the country of origin if they are detected. All domestic workers and illegal immigrants are employed.

Perfectly competitive firms produce domestic output y by using labor supplied by native households, l, along with labor provided by illegal immigrants, l_I. l and l_I are both expressed in terms of hours worked. The production technology is given by $y = F(l + l_I)$, where $F(\,\cdot\,)$ is continuously twice differentiable, strictly increasing and concave. Without any loss of generality, it is assumed that domestic labor and labor of illegal immigrants are perfectly substitutable in production.

Labor of domestic residents is paid at the competitive wage w, while labor of illegal immigrants is paid at the exploitative wage $w_I < w$; the exploitative wage exists because hiring illegal immigrants is risky for firms.[10]

Maximum profit of firms requires that

$$F'(l + l_I) = w, \tag{1a}$$

$$w_I = \beta w, \tag{1b}$$

where $\beta \in (0, 1]$ is the fixed immigrant wage to native wage ratio. β measures of the effectiveness of policies against illegal immigrants.

By using (1), firms' profits Π, which are positive, can be expressed as

$$\Pi = F(l + l_I) - (l + \beta l_I)F'(l + l_I) > 0. \tag{2}$$

Domestic consumers, whose number is constant over time, maximize the integral utility

[10] Once discovered, firms that employ illegal workers have to pay a fine, while immigrants are expatriated. See, for example, Hazari and Sgro (2003), Moy and Yip (2006), and Palivos (2009). As shown by Palivos (2009), the condition $w_I < w$ can be obtained from a model in which firms that employ unauthorized immigrants have to pay a penalty when they are discovered hiring workers illegally.

$$\int_0^\infty U(c,x)\mathrm{e}^{-\rho t}dt, \tag{3}$$

where c is consumption, x leisure, and ρ the fixed rate of time preference. The instantaneous utility function $U(\,\cdot\,)$ is strictly increasing and concave in its arguments.

Native consumers accumulate wealth by holding government bonds, d, and real money balances, m. The real rate of return earned by holding government bonds is r. The opportunity cost of holding money —i.e., the nominal interest rate— is $i = r + \pi$, where π is the inflation rate. Domestic households have to pay transaction costs $s(v)$ to consume one unit of the consumption good, where v denotes the consumption to money ratio; that is,

$$v = \frac{c}{m}. \tag{4}$$

The transaction cost function $s(\,\cdot\,)$, which is continuous and twice differentiable, satisfies the following properties:[11]

i) $s(v) \geqslant 0$ for $v \geqslant 0$;

ii) a critical level $\widetilde{v}$ of v exists, which corresponds to the satiation level of real money balances, for which $s(\widetilde{v}) = s'(\widetilde{v}) = 0$;

iii) $(v - \widetilde{v})s'(v) > 0$ for $v \neq \widetilde{v}$;

iv) $s'' > 0$ for $v \geqslant 0$.

The domestic residents' flow budget constraint is given by

$$c[1 + s(v)] + \dot{d} + \dot{m} = r(d + m) + (1 - \tau)(wl + \Pi) - im, \tag{5}$$

where τ is a proportional income tax rate.

The time endowment of domestic residents, which is normalized to one, being fixed, is used for either working or consuming leisure; that is,

$$1 = x + l. \tag{6}$$

The maximization of (3) subject to (4), (5) and (6) yields the following first-order conditions

[11] See Feenstra (1986), and Schmitt-Grohè and Uribe (2010 and 2012).

$$U_c = \lambda(1 + s + vs'), \tag{7a}$$

$$U_x = (1 - \tau)w\lambda, \tag{7b}$$

$$v^2 s' = i, \tag{7c}$$

$$-\dot{\lambda} + \lambda\rho = \lambda r, \tag{7d}$$

where λ is the shadow value of domestic resident wealth.[12]

Foreign agents, that enter and work illegally in the host country, accumulate wealth by holding real money balances m_I, necessary for purchasing domestic consumption goods, and foreign bonds (denominated in domestic currency) b in their portfolios. Because of perfect capital mobility, the real rate of return earned by holding foreign bonds, expressed in terms of the domestic numeraire, is r.

The illegal immigrants' flow budget constraint in aggregate terms is[13]

$$c_I[1 + q(v_I)] + \dot{m}_I + \dot{b} = r(m_I + b) + w_I l_I - i m_I, \tag{8}$$

where c_I denotes illegal immigrants' consumption, $q(\ \cdot\ )$ —which satisfies the same qualitative properties of $s(\ \cdot\ )$— their transaction costs function, and m_I their money holdings; v_I is the consumption-based money velocity of illegal immigrants, defined as

$$v_I = \frac{c_I}{m_I}. \tag{9}$$

[12] Equations (7) are fairly standard. After using (7a), equation (7b) can be written as $\dfrac{U_x}{U_c} = \dfrac{(1-\tau_l)w}{(1+s+vs')}$. Such an equation states that the marginal rate of substitution of consumption for leisure must equal the opportunity cost of leisure in terms of consumption; this opportunity cost is equal to the after-tax wage divided by one plus the marginal cost of consumption. Equation (7c) is the implicit demand for real money balances. The money demand in explicit terms is: $m = cL(i)$, with $L' = -\frac{1}{(2vs' + v^2 s'')v^2} < 0$.

[13] l_I, total hours worked by foreign workers, is proportional to the number of illegal immigrants if each immigrant works the same fixed number of hours as the others.

It is assumed that unauthorized immigrants do not pay income taxes because of their illegal condition.

Illegal immigrants maximize their utility function subject to (8) and (9). We are only interested in their money demand function, because this is an element that the Ramsey planner has to take into account when choosing the optimal monetary policy. Therefore, only the first-order condition of the illegal immigrant utility maximization with respect to v_I, which is given by

$$v_I^2 q' = i, \tag{10}$$

has to be considered in the Ramsey problem.

Since unauthorized immigrants may use domestic money more intensively than native consumers, because they operate in the illegal sphere of the economy and aim at remaining concealed,[14] we assume that $q'(v_0) \geqslant s'(v_0)$.[15] This implies that $v \geqslant v_I$ for every i.[16]

The government finances its budget deficit by issuing government debt and money, whose total stock in real terms is $m + m_I$. The government budget constraint is given by

$$\dot{d} + \dot{m} + \dot{m}_I = r(d + m + m_I) + g - \tau(wl + \Pi) - i(m + m_I), \tag{11}$$

[14] As illegal immigration is associated with a sort of informal sector, such a hypothesis is consistent with the assumption adopted by Nicolini (1998) that the informal economy is more currency intensive than the official one.

[15] By keeping the function $q(\cdot)$ potentially separated from the function $s(\cdot)$, we can pay attention to the differential role played by v_I with respect to v for the optimal monetary policy.

[16] Assuming the same functional form for the transaction costs technology as Schmitt-Grohè and Uribe (2010 and 2012), we have that

$$s(v) = Av + B/v - 2\sqrt{AB},$$

and

$$q(v_I) = A_I v_I + B/v_I - 2\sqrt{A_I B},$$

where $A > 0$, $B > 0$ and $A_I \geqslant A$ are parameters. The satiation levels of the consumption-money ratio for nationals and illegal immigrants are $\widetilde{v} = \sqrt{B/A}$ and $\widetilde{v}_I = \sqrt{B/A_I}$ (with $\widetilde{v} \geqslant \widetilde{v}_I$), respectively.

where g denotes government spending, assumed to be exogenous.

The resource constraint states that domestic output plus interest income earned by illegal immigrants by holding foreign bonds is equal to total consumption costs of domestic consumers and foreign workers plus government spending plus the rate of accumulation of foreign bonds; that is,

$$F(l + l_I) + rb = c[1 + s(v)] + c_I[1 + q(v_I)] + g + \dot{b}. \tag{12}$$

3 Ramsey monetary policy with income taxes only

Only second-best monetary policy is analyzed. The first-best monetary policy, obtained when distortionary taxation is absent and there are only lump-sum taxes, is the Friedman rule if both domestic consumers and illegal immigrants are compensated for the inflation tax.[17]

The second-best monetary policy is obtained by maximizing the social welfare function, given by the utility of the representative domestic consumer, subject to the competitive equilibrium with distortionary taxes and the constraint that a given flow of government spending has to be financed.

As illegal immigrants use domestic money for making transactions in the host country, the Ramsey planner —that does not consider their welfare, because of their unauthorized status— has to take indirectly into account, when choosing the optimal inflation rate, their money demand through the feasibility constraint.

The analysis of the efficient policy is based on the implementability constraint of nationals. This is obtained from the domestic consumers' intertemporal budget constraint after expressing prices and taxes in terms of quantities through the marginal efficiency conditions (7).

The implementability constraint of the domestic consumers is derived as follows. By integrating the flow budget constraint (5) forward and incorporating the condition preventing 'Ponzi games' —that is, $\lim_{t\to\infty} (d + m)$ $\mathrm{e}^{-\int_0^t r du}$=0— yields the residents' intertemporal budget constraint

[17] The analysis of the first-best monetary policy is provided in the Appendix.

$$\int_0^\infty \{c[1+s(v)] + im - (1-\tau)(wl+\Pi)\}\,\mathrm{e}^{-\int_0^t r du} dt = 0, \qquad (5')$$

where we have set $d_0 + m_0 = 0$ (d_0 and m_0 are government debt and real money balances at $t = 0$).

By integrating (7d), we get

$$e^{-\int_0^t r du} = \frac{\lambda}{\lambda_0} e^{-\rho t}, \qquad (7d')$$

where λ_0 is λ at time 0.

Substituting (2), (7a), (7b), (7c) and (7d') into (5'), one obtains, after rearranging, the implementability constraint; that is,

$$\int_0^\infty \{cU_c - U_x \frac{[F(l+l_I) - \beta F' l_I]}{F'}\} \mathrm{e}^{-\rho t} dt = 0. \qquad (13)$$

A second constraint faced by the Ramsey planner when setting the optimal monetary policy is the feasibility constraint expressed in terms of v, which is obtained as follows. After using (1), the combination of the flow budget constraint of illegal immigrants (8) with (12) allows us to express the feasibility constraint as

$$\dot{m}_I = c[1+s(v)] + g + \beta F' l_I - F(l+l_I) - (i-r)m_I. \qquad (14)$$

By equating (7c) and (10) for i, and solving for v_I, yields

$$v_I = V(v), \qquad (15)$$

where $V' = \dfrac{(2vs' + v^2 s'')}{(2v_I q' + v_I^2 q'')} > 0$.

Using (9) and (15), the money demand of illegal immigrants can be expressed as $m_I = \dfrac{c_I}{V(v)}$. The total differentiation of this equation yields $\dot{m}_I = \dfrac{\dot{c}_I}{v_I} - \dfrac{c_I V'}{v_I^2}\dot{v}$. Plugging $\dot{m}_I$ from such a relationship into (14), making use of (7c) and (7d) to eliminate i and r, respectively, and rearranging, we obtain

$$\dot{v} = \frac{v_I^2}{c_I V'} \left\{ \frac{\dot{c}_I}{v_I} - c[1 + s(v)] - g - \beta F' l_I + F(l + l_I) + \frac{c_I}{v_I}(v^2 s' + \frac{z}{\lambda} - \rho) \right\}, \tag{16}$$

where $z = \dot{\lambda}$. Equation (16) represents the feasibility constraint expressed in terms of v.

The social planner chooses the Ramsey allocation by maximizing the domestic consumer utility integral (3) subject to the implementability constraint (13) and the feasibility constraint (16), once the relationships $z = \dot{\lambda}$ and $\lambda(1 + s + vs') = U_c$ are taken into account. Moreover, as the nominal interest rate cannot be negative, also the inequality $v \geqslant \widetilde{v}$ has to be imposed. The variables chosen by the social planner are: c, l, l_I, v, z and λ.

The optimal tax structure can be summarized as follows:

Proposition 1 *In an infinitely lived monetary model with transaction costs and illegal immigrants, that demand domestic money for making transactions and consume in the host country, second-best efficiency prescribes to tax real money balances in addition to domestic consumers' income when consumption taxation is not available.*

Proof. The first-order conditions of the Ramsey problem with respect to v, z and λ, and the Kuhn-Tucker conditions regarding the inequality constraint $v \geqslant \widetilde{v}$, are given by

$$-\dot{\Gamma} + \rho\Gamma = \Gamma \frac{v_I^2}{c_I V'} \left[-cs' + \frac{c_I}{v_I}(2vs' + v^2 s'') - (v^2 s' + \frac{z}{\lambda} - \rho)\frac{c_I}{v_I^2} V' \right] +$$

$$+\Sigma\lambda(2s' + vs'') + \Xi, \tag{17a}$$

$$\frac{\Gamma v_I}{\lambda V'} = -\Delta, \tag{17b}$$

$$-\dot{\Delta} + \rho\Delta = -\frac{\Gamma v_I z}{\lambda^2 V'} + \Sigma(1 + s + vs'), \tag{17c}$$

$$\Xi(v-\widetilde{v})=0,\ \Xi\geqslant 0,\ v-\widetilde{v}\geqslant 0, \tag{17d}$$

where Γ, Δ and Σ denote the Lagrange multipliers on the constraints (16), $z=\dot{\lambda}$ and $\lambda(1+s+vs')=U_c$, respectively. Ξ is the Kuhn-Tucker multiplier on the inequality constraint $v\geqslant\widetilde{v}$.

In the long-run, plugging (17b) into (17c) for Δ and rearranging, we get

$$\Sigma\lambda=-\frac{\Gamma\rho v_I}{(1+s+vs')V'}. \tag{18}$$

Using such an expression into the steady state version of (17a) yields

$$\Gamma\left(\frac{v_I^2 c}{c_I V'}+v^2\right)s'=\frac{\Gamma v_I}{V'}(2s'+vs'')\left[v-\frac{\rho}{(1+s+vs')}\right]+\Xi. \tag{19}$$

We demonstrate that the Friedman rule is not optimal by showing that (19) is contradicted when the nominal interest rate is zero. In fact, if $v=\widetilde{v}$ —and hence $s=s'=0$ and $v_I=V(\widetilde{v}\ )=\widetilde{v}\ _I$— equation (19) becomes

$$\frac{\Gamma\widetilde{v_I}\widetilde{v}s''}{V'}(\widetilde{v}-\rho)+\Xi=0. \tag{19'}$$

As plausibly $\widetilde{v}>\rho$,[18] (19') implies that $\Xi<0$ because $\Gamma>0$, $V'>0$ and $s''>0$.[19] But since the Kuhn-Tucker condition (17d) is violated, v cannot be equal to $\widetilde{v}$; that is, the Friedman rule cannot be optimal.

[18] This assumption is also adopted by Schmitt-Grohè and Uribe (2010 and 2012).

[19] The fact that $\Gamma>0$ can be demonstrated as follows. The first-order condition of the Ramsey problem with respect to c is

$$U_c[1+\Phi(1+\eta_c)]=\frac{\Gamma v_I^2(1+s)}{c_I V'}+\Sigma U_{cc},$$

where $\eta_c=\frac{cU_{cc}}{U_c}-\frac{U_{xc}(F-\beta F' l_I)}{F'}$. After using (18), such an equation can be written as

$$\Gamma=\frac{U_c[1+\Phi(1+\eta_c)]V'}{v_I[\frac{v_I(1+s)}{c_I}-\frac{\rho U_{cc}}{U_c}]}.$$

Therefore, $\Gamma>0$ as the marginal pseudo-utility of consumption is positive, as $\Phi>0$ (because of positive income taxation) and $1+\Phi(1+\eta_c)>0$.

Therefore, it must alternatively be that $v > \widetilde{v}$ and $\Xi = 0$.[20] $\square$

When only income taxation is available, the presence of illegal immigrants implies that it is Ramsey efficient to raise revenues by taxing money holdings (in addition to income of domestic consumers) even if this is costly for those who matter for the social planner (i.e., domestic consumers). As there are practical restrictions on the capacity of the planner to tax unauthorized immigrants, because of their illegality, the inflation tax represents a way of collecting resources from them.[21]

Analytically speaking, the departure from the Friedman rule is due to the fact that, because of the inflation tax, the illegal immigrants' money demand enters the feasibility constraint. Similarly to Schmitt-Grohè and Uribe (2010 and 2012), this fact undermines the optimality of the zero inflation tax result.[22]

4 Ramsey monetary policy with income and consumption taxes

Suppose that a proportional tax on consumption (whose rate is τ_c), paid by both domestic consumers and illegal immigrants when they acquire consumption goods, is introduced. In this case, the domestic consumers' cost of consumption, which has to be considered in their flow budget constraint (5), is $c[1 + \tau_c + s(v)]$, while the consumption cost for unauthorized immigrants,

[20]After using $\Xi = 0$ and plugging (7c) into (19), the optimal inflation tax is obtained; that is,

$$i^* = \frac{v^2 v_I s''[v(1+s+vs') - \rho]}{(1+s+vs')\left[\frac{v_I^2 c}{v^2 c_I} + V' - \frac{2v_I}{v} + \frac{2\rho v_I}{v^2(1+s+vs')}\right]} > 0.$$

[21]Such a result is due to the well-established principle of public finance that also intermediate goods (like real money balances) should be taxed when the tax code is not sufficiently rich. See Munk (1980), and Chari and Kehoe (1999).

[22]Notice instead that the second-best invalidity of the Friedman rule hinges in general on the fact that real money balances enter (through a derivative of the transaction cost technology) either the implementability constraint or additional constraints faced by the planner, which private agents do not face. See Petrucci (2011).

that has to be included in their budget constraint (8), is $c_I[1+\tau_c+q(v_I)]$.

The only change that the presence of a consumption tax implies for the first-order conditions (7) is the following

$$U_c = \lambda(1+\tau_c+s+vs'), \tag{7a'}$$

which replaces (7a).

In the government budget constraint (11), the term $\tau_c(c+c_I)$ has to be included on the revenue side.

The second-best tax problem is the same as in the case in which only income taxation is available with three differences. One is that (7a') instead of (7a) has to be considered within the constraints of the Ramsey problem, one is that the feasibility constraint (16) has to include the term $\tau_c c_I$ inside the curly brackets of the right-hand side, and one is that τ_c is endogenosuly chosen.

The conceptual characterization of the efficient tax policy is:

Proposition 2 *In an immortal monetary economy with costly transactions and illegal immigration, the optimal inflation tax strictly depends on the consumption-money ratios of nationals and unauthorized immigrants when income and consumption taxes are available. If both agents have the same consumption-money ratio, the Friedman rule is second-best optimal. If instead the consumption-money ratio of illegal immigrants is lower than the consumption-money ratio of nationals, it is efficient to deviate from the Friedman rule.*

Proof. The relevant first-order conditions for the second-best optimal monetary policy are given by (17a), (17b), (17d) and

$$-\dot{\Delta}+\rho\Delta = -\frac{\Gamma v_I z}{\lambda^2 V'}+\Sigma(1+\tau_c+s+vs'), \tag{17c'}$$

$$\frac{\Gamma v_I^2}{V'} = -\Sigma\lambda. \tag{17e}$$

Plugging (17e) into the long-run version of (17a) yields

$$\Gamma\left(\frac{v_I^2 c}{c_I V'}+v^2\right)s'=\frac{\Gamma v_I}{V'}(2s'+vs'')(v-v_I)+\Xi. \tag{20}$$

Two cases are possible: i) $v=v_I$; and ii) $v>v_I$. Consider each case in turn.

i) $v=v_I$[23]

Suppose that $v>\widetilde{v}$. Now equation (20) implies that $\Xi>0$ as $s'(v)>0$ and $\Gamma>0$. But this is impossible, since the Kuhn-Tucker condition (17d) requires that $\Xi=0$ when $v>\widetilde{v}$. Therefore, setting a positive nominal interest rate is nonoptimal.

ii) $v>v_I$[24]

Suppose that $v=\widetilde{v}$ and hence $v_I=V(\widetilde{v})=\widetilde{v}_I$. In this case, equation (20) becomes

$$\frac{\Gamma\widetilde{v}_I\widetilde{v}s''}{V'}(\widetilde{v}-\widetilde{v}_I)+\Xi=0. \tag{20'}$$

This implies that $\Xi<0$. But then, as the Kuhn-Tucker condition (17d) is contradicted, v and v_I cannot be equal to $\widetilde{v}$ and $\widetilde{v}_I$, respectively.

Therefore, we must instead have $v>\widetilde{v}$ (and hence $v_I>\widetilde{v}_I$) as well as $\Xi=0$. Thus, when $v>v_I$, deviating from the Friedman rule is optimal.[25] □

In a model with illegal immigrants, whose income cannot be taxed, the consideration of consumption taxation may remove the presence of tax limitations. When $v=v_I$, it is optimal to set the implicit cost of holding money to zero as illegal immigrants contribute to the revenue collection through consumption taxation in the same relative way as domestic consumers. In this case, although the consumption tax and the inflation tax work similarly, the inflation tax is a less efficient way to obtain revenues.

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[23] In this case $s(\,\cdot\,)=q(\,\cdot\,)$. $A=A_I$ if the transaction cost functional form of footnote 15 is employed.

[24] Such an inequality is satisfied if $q'(v_0)>s'(v_0)$ for $v_0>0$. With the transaction technologies of footnote 15, this case occurs when $A>A_I$.

[25] In this case, the optimal inflation tax, obtained from (20), after employing (7c), is

$$i^*=\frac{v^2 c_I v_I s''(v-v_I)}{[v_I^2 c+V' v^2 c_I-2v_I c_I(v-v_I)]}>0.$$

When $v > v_I$, the consumption tax and the inflation tax are no longer equivalent. In fact, the consumption tax hits relatively more domestic consumers, while the inflation tax hits relatively more illegal immigrants. Therefore, it is optimal to set a positive nominal interest rate alongside a positive consumption tax for collecting a given amount of revenue efficiently and distributing the tax burden across legal and illegal agents.

5 Concluding remarks

This paper has investigated the issue of the optimal inflation tax in a transaction costs model with illegal immigration. What makes the economic environment studied here interesting for the optimal monetary policy is that illegal immigrants demand domestic currency and consume in the host country, but their income is not taxed and their welfare does not enter the planner's objective function. These features of the economic setup lead to the fact that collecting revenues from seignorage can be second-best optimal. In fact, by extracting resources from illegal immigrants through the inflation tax, the Ramsey planner may increase consumption of those whose welfare matters for him.

The specific results of the paper are the following. If only income taxation is available, the optimal inflation tax should be positive, as it represents an indirect way to tax illegal immigrants, who cannot alternatively be taxed. In this case, the incompleteness of the tax system is (once again) at the root of the invalidity of the Friedman (1969) monetary policy prescription.

When instead also a consumption tax is at the disposal of the planner and can be optimally chosen, optimality calls for either the Friedman (1969) rule — when domestic residents and illegal aliens have the same consumption-money ratio— or the Phelps (1973) rule —when the consumption-money ratio of domestic residents is higher than the one of illegal immigrants. In fact, when domestic consumers and illegal immigrants face the same transaction costs, and hence are affected by consumption taxation equally, the consumption tax is more efficient than the inflation tax. When instead consumption taxation hits nationals relatively more than unauthorized aliens (that is, the consumption-money ratio of domestic consumers is higher than the one of illegal immigrants), the inflation tax and the consumption tax are both required to collect a given amount of revenue, and distribute the tax burden efficiently on nationals and illegal immigrants.

Appendix: First-best monetary policy

In this Appendix, the first-best monetary policy is studied.

Assume that there are no distortionary taxes and that only lump-sum taxation is used to balance the government budget. When both domestic consumers and illegal immigrants are compensated for the inflation tax, the optimal monetary policy is the Friedman rule.[26]

In this case, the optimal monetary policy is obtained by maximizing the social welfare function subject to the proper feasibility constraint (derived below) and the inequality $v \geqslant \widetilde{v}$.

The feasibility constraint for the first-best monetary policy, expressed in terms of v, is obtained as follows. By using the flow budget constraint of illegal immigrants together with (12) —under the hypothesis that such agents are lump-sum compensated for the inflation — the feasibility constraint can be written as

$$\dot{m}_I = c[1 + s(v)] + g + \beta F' l_I - F(l + l_I). \tag{14'}$$

After differentiating the illegal immigrants' money demand and using (15), from (14') we obtain

$$\dot{v} = \frac{v_I^2}{c_I V'} \left\{ \frac{\dot{c}_I}{v_I} - c[1 + s(v)] - g - \beta F' l_I + F(l + l_I) \right\}. \tag{16'}$$

Equation (16') is the feasibility constraint expressed in terms of v under the assumption that illegal immigrants are compensated for the inflation tax.

[26] The first-best monetary policy is the Phelps (1973) rule if illegal immigrants are not compensated for the inflation tax as seignorage, obtained from domestic residents and unauthorized aliens, is fully lump-sum distributed to nationals. In this circumstance, the optimal monetary policy is obtained by maximizing the utility integral (3) subject to the feasibility constraint (16), the relationships $z = \dot{\lambda}$ and $\lambda(1 + s + vs') = U_c$, and the inequality $v \geqslant \widetilde{v}$. As the first-order conditions of this problem, relevant for our purpose, are given by equations (17), it can be easily shown that the Friedman rule is nonoptimal, by using the demonstration of Section 3.

The social planner chooses the first-best allocation by maximizing the utility functional (3) subject to (16') and the restriction $v \geqslant \widetilde{v}$. The variables chosen by the social planner are: c, l, l_I and v.

The first-order condition of the optimal monetary policy with respect to v and the Kuhn-Tucker condition are

$$-\dot{\Gamma} + \rho\Gamma = -\Gamma\frac{v_I^2 cs'}{c_I V'} + \Xi, \tag{21a}$$

$$\Xi(v - \widetilde{v}) = 0,\ \Xi \geqslant 0,\ v - \widetilde{v} \geqslant 0. \tag{21b}$$

In the long-run, (21a) becomes

$$\rho\Gamma = -\Gamma\frac{v_I^2 cs'}{c_I V'} + \Xi. \tag{21a'}$$

If $v > \widetilde{v}$ and hence $\Xi = 0$, (21a') would imply that $s' = i^*/v^2 < 0$, thus contradicting the restriction of a nonnegative nominal interest rate. If instead $v = \widetilde{v}$, (21a') implies, consistently with the Kuhn-Tucker conditions, that $\Xi = \rho\Gamma > 0$. Therefore, the Friedman rule is first-best optimal.

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