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On Robust Asymmetric Equilibria in Asymmetric R&D-Driven Growth Economies*

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Abstract

In R&D-driven growth models with asymmetric fundamentals the steady-state equilibrium R&D investments are industry-specific and they are such that R&D returns are equalized across industries. Return equalization, however, makes investors indifferent as to where to target research and, hence, the problem of allocation of R&D investments across industries is indeterminate. Agents' indifference creates an ambiguous investment scenario. We assume that agents hold "ambiguous" beliefs on the per-industry profitability of their R&D investments. Investors' aversion towards ambiguity eliminates the indeterminacy of the investment problem. In particular, the asymmetric return-equalizing equilibrium is robust against a however small degree of investors' ambiguity aversion.

Keywords: R&D driven growth models, ambiguity, $\varepsilon-$ contamination.

JEL Classification: 032, 041, D81.

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1 Introduction

R&D-driven growth models focus on the role of technical change as the main source of economic growth. In this class of models technical change is "endogenous", as it is the outcome of R&D investment decisions taken by profit maximizing firms. The innovation process can either be captured as an increasing number of available goods (*horizontal innovation*, Romer 1990) or as a process aimed at improving the quality, or the productivity, of existing goods (*vertical innovation*, see for instance Anant, Dinopoulos and Segerstrom (1990), Grossman and Helpman (1991), Aghion and Howitt (1992)). In either case, by focusing on the relationship between the amount of R&D investments and the macroeconomic performance, these models have played a major role in shaping the recent policy debate over the opportunity of intervening via subsidies (or taxes) to R&D expenditures to foster technological progress and thus economic growth.

R&D-driven growth models succeeded in making technical change endogenous by introducing monopolistic competition and (aggregate) increasing returns. Accordingly, market equilibrium and the amount of resources devoted to R&D are not optimal. In particular, while it is almost pacific that an increasing amount of R&D efforts is growth-enhancing,[1] its effect on the economy's welfare is instead ambiguous. Indeed, within the family of vertical R&D models, the amount of R&D efforts that the market commands may be higher than the one which maximizes aggregate welfare (because of the so called "business stealing" effect).

The debate over the policy implications of these models has continued over the years and has taken several directions. For instance, the discussion on the existence of "scale effects", originated by Jones's (1995 a,b) seminal papers, has powerful policy implications, in terms of whether or not an increase in R&D investments raises the economy's growth rate temporarily or permanently. The relation between the optimal (possibly negative) subsidy and the size of innovation has also been thoroughly investigated (see Grossman and Helpman (1991), Segerstrom (1998) and Li (2003) among others).

Recent contributions have attempted to further enrich the picture by investigating the implications of extending vertical R&D-driven growth models to encompass economies with asymmetric fundamentals. The standard vertical models present a symmetric structure, in the sense all industries share the same cost and demand condi-

[1] A notable exception is Segerstrom's (2000) 'paradox of growth'.

tions. Giordani and Zamparelli (2008) and Cozzi and Impullitti (2009) independently developed models in which industries are asymmetric as they may differ from each other in terms of, for instance, market size or innovation potential. The former show that an industrial policy which, say, redistributes resources towards the more promising industries raises the long-run growth rate and the social welfare of the economy. The latter prove that a shift in the composition of government spending favoring the most innovative firms increases innovation, population's education attainment and the skill premium.

With respect to the vertical R&D models, the overall policy debate relies on a comparative statics analysis around a unique steady-state equilibrium, whose main feature is that the cross-industry distribution of R&D investments is such that R&D returns are equalized across industries (as we will clarify below). Policy implications are then valid to the extent this equilibrium is uniquely pinned down. This, however, may well not be the case in the class of both symmetric and asymmetric models.

In the standard symmetric models the (return-equalizing) steady state equilibrium is symmetric, in the sense that an equal amount of R&D resources is devoted to each existing industry. The focus on symmetric equilibrium relies on two assumptions. The first is that industries are indistinguishable in terms of fundamentals. The second is concerned with the role played by *expectations* and needs more discussion.

Since the innovation process is vertical, the monopolistic position of an entrepreneur patenting an innovation has a *temporary* nature, as it only lasts until the next improvement in the same industry occurs. This distinctive feature of the vertical innovation literature, usually referred to as "creative destruction" (Schumpeter, 1942 [1975]), is responsible for the role of expectations on *future* R&D investment decisions in determining the amount and the distribution across industries of *current* R&D investment. Since investors anticipate that their monopolistic position will only last up to the next innovation in their product line, their incentive to invest in R&D in a particular industry is negatively affected by the future amount of R&D investment expected in that industry. In turn, in order to focus on symmetric R&D investment, the additional assumption of *symmetric expectations* needs to be made. Only under the joint hypothesis of symmetric expectations and symmetric fundamentals, investors are indifferent as to which industry they target, and hence the model may focus on a symmetric solution to the allocation of R&D efforts (see Grossman and Helpman, 1991, p.47). Indeed, Cozzi (2005, 2007) shows the existence of multiple asymmetric equilibria triggered by self-fulfilling *asymmetric expectations*.

These two assumptions (symmetric fundamentals and expectations) however, do not univocally pin down the symmetric equilibrium. Equal expected profitability across industries leaves the investor *indifferent* as to which industry to select when deciding the allocation of current R&D efforts. The allocation problem is indeterminate. This indeterminacy in the distribution of R&D investment may generate multiple asymmetric equilibria, analogous to those identified by Cozzi (2005, 2007), each characterized by a different balanced growth path.

A similar issue arises in an asymmetric theoretical framework. In Giordani and Zamparelli (2008) and Cozzi and Impullitti (2009) an R&D growth model with asymmetric fundamentals is developed, where the resulting steady state equilibrium is asymmetric. In particular, the balanced growth path equilibrium is characterized by an asymmetric configuration of R&D investments capable of equalizing R&D returns across industries. As in the standard symmetric case, equalization of returns leaves the agent *indifferent* as to which industry to invest in. As a result, the *asymmetric equilibrium* is not uniquely pinned down.

In a recent paper Cozzi et Al. (2007) solve the indeterminacy of the symmetric equilibrium. They prove that the symmetric equilibrium is the only rational expectations equilibrium robust to a however small "degree" of investors' ambiguity aversion in the evaluation of R&D returns. In a similar vein, in this paper we generalize this result, by making the focus on the (return equalizing) asymmetric equilibrium compelling. Since the policy implications of the asymmetric framework are based on the return-equalizing equilibrium, establishing its robustness improves our confidence in the validity of those policy implications.

Our basic idea is that the agents' indifference - arising from the equalization of R&D returns across industries - gives them in principle the possibility of adopting a whatever (even randomly chosen) investment strategy. This makes these agents highly uncertain about the configuration of future R&D investment, since that configuration is the result of a decision problem analogous to the one they are currently facing. As a consequence, we assume that the agent's beliefs on the future (per industry) distribution of R&D investments are characterized by uncertainty (or *ambiguity*), in the sense that information about that distribution is too imprecise to be represented by a (single additive) probability measure. To formalize a taste for uncertainty we follow a mutiple prior approach in the spirit of Gilboa and Schmeidler (1989) and, in particular, we use an "$\varepsilon-$contamination of confidence" argument, axiomatized by Nishimura and Ozaki (2006). We show that a however small "degree" of uncertainty in the expectations of the future investment's allocation (an "ε−contamination of confidence") eliminates agents' indifference and makes the configuration where R&D returns are equalized across industries arise as the unique rational expectations equilibrium.

The rest of the paper is organized as follows. In Section 2 we introduce the model. In Section 3 we explain the core of our argument, and state our main result. Proof and other technicalities are relegated to Appendices A and B.

2 The Model

In this section we build a vertical innovation (or "quality ladder") growth model with asymmetric fundamentals much in the spirit of Giordani and Zamparelli (2008). Let us assume a continuum of industries producing final goods indexed by $\omega \in [0, 1]$. In each industry firms are distinguished by the quality index j of the goods they supply, with the quality of their goods being increasing in the integer j. At time $t = 0$ in each industry some firm knows how to produce a $j = 0$ quality product and no other firm can offer a better one. In order to develop higher quality versions of any product firms engage in R&D races. The winner of an R&D race becomes the sole producer of a good whose quality is one step ahead of the previous quality leader.

There exists a fixed number of dynastic households (normalized to one) whose members grow at constant rate $n > 0$. Each member shares the same intertemporally additively separable utility $\log u(t)$ and is endowed with a unit of labor she supplies inelastically. Therefore each household chooses her optimal consumption path by maximizing the discounted utility

$$U \equiv \int_0^\infty L(0)e^{-(\rho-n)t} \log u(t)dt \tag{1}$$

where $L(0) \equiv 1$ is the initial population and $\rho > n$ is the common rate of time preferences.

The instantaneous utility function is a logarithmic Cobb-Douglas. We let the utility weights ($\alpha(\omega)$) vary across industries, so as to represent a possible heterogeneity of consumers' preferences among the set of commodities. As the $\alpha(\omega)$'s represent the relative weights of the goods in the utility function, we can normalize them in such a way that $\int_0^1 \alpha(\omega)d\omega = 1$. If we define $\lambda(\omega)$ as the size of quality improvements (the so-called "quality jump"), assumed to be industry-specific to allow for asymmetry in the technical evolution of each line, $j^{\max}(\omega,t)$ as the highest quality reached by product ω at time t, and $d(j,\omega,t)$ as the consumption of product ω of quality j at time t, then the instantaneous utility function can be written as

$$\log u(t) \equiv \int_0^1 \alpha(\omega) \log \sum_{j=0}^{j^{\max}(\omega,t)} \lambda^j(\omega) d(j,\omega,t) d\omega, \tag{2}$$

and the static maximization problem can be represented as

$$\begin{aligned} &\max_d \int_0^1 \alpha(\omega) \log \sum_{j=0}^{j^{\max}(\omega,t)} \lambda^j(\omega) d(j,\omega,t) d\omega \\ s.t.\ \ E(t) \ &= \int_0^1 \left[\sum_{j=0}^{j^{\max}(\omega,t)} p(j,\omega,t) d(j,\omega,t) \right] d\omega \end{aligned}$$

where $p(j,\omega,t)$ denotes the price of product ω of quality j at time t, and $E(t)$ is the total expenditure at time t.

At each point in time consumers maximize static utility by spreading their expenditure across industries proportionally to the utility contribution of each product line ($\alpha(\omega)$), and by only purchasing in each line the product with the lowest price per unit of quality. As usual in quality-ladder models with Bertrand competition in the manufacturing sector, this product is the one indexed by $j^{\max}(\omega,t)$. As a result, the individual static demand functions are

$$d(j,\omega,t) = \begin{cases} \dfrac{\alpha(\omega)E(t)}{p(j,\omega,t)} & \text{for } j = j^{\max}(\omega,t) \\ 0 & \text{otherwise} \end{cases} \tag{3}$$

Moreover, since the only $j^{\max}(\omega,t)$ quality product is actually purchased, in what follows it will be

$$\sum_{j=0}^{j^{\max}(\omega,t)} \lambda^j(\omega) = \lambda^{j^{\max}(\omega,t)}(\omega).$$

Substituting (3) into (2) and (2) into (1) we get the intertemporal maximum problem as

$$\begin{aligned} \max_E U \ &= \int_0^\infty e^{-(\rho-n)t} [\log E(t) + \int_0^1 \alpha(\omega) [\log \alpha(\omega) + \log [\lambda(\omega)]^{j^{\max}(\omega,t)} \\ &\quad - \log p(j,\omega,t)] d\omega] dt \end{aligned}$$

$$\text{s.t.} \int_0^{\infty} e^{-\int_0^t [r(s)-n]ds} E(t)dt \leq W(0),$$

where $r(s)$ is the instantaneous interest rate at time s and $W(0)$ is the present value of the stream of incomes plus the value of initial wealth at time $t = 0$. The solution to this problem obeys the differential equation

$$\frac{\dot{E}(t)}{E(t)} = r(t) - \rho.$$

Each good is produced by only employing labor through a constant return to scale technology: in order to produce one unit of good ω firms hire l_ω units of labor regardless of quality. The Bertrand competition assumption implies that the quality leader monopolizes her relative market and that the limit price she can charge is $p\left[j^{\max}(\omega, t), \omega, t\right] = \lambda(\omega) w l_\omega$. Thus the profit flows in each industry are

$$\pi(\omega, t) = \frac{\lambda(\omega) - 1}{\lambda(\omega)} \alpha(\omega) E(t) L(t).$$

Firms can engage in R&D to develop better versions of the existing products in order to displace the current monopolists. We assume free entry and perfect competition in each R&D race. Firms employ labor and produce, through a constant returns technology, a Poisson arrival rate of innovation in the product line they target. The R&D technology is industry-specific. In particular, any firm hiring l_k units of labor in industry ω at time t acquires the instantaneous probability of innovating $A(\omega) l_k / X(\omega, t)$, where $X(\omega, t)$ is the R&D difficulty index. Since independent Poisson processes are additive, the specification of the innovation process implies that the industry-wide instantaneous probability of innovation (or research intensity) is

$$\frac{A(\omega) L_I(\omega, t)}{X(\omega, t)} \equiv i(\omega, t)$$

where $L_I(\omega, t) = \sum_k l_k(\omega, t) dk$. As R&D proceeds, its difficulty index $X(\omega, t)$ is supposed to increase over time in order to rule out the "scale effect" (Jones, 1995a,b), that is, to rule out explosive growth in the presence of a growing population. With reference to Segerstrom (1998), we model the increasing complexity hypothesis according to what is usually called 'TEG specification':[2]

$$\frac{\dot{X}(\omega,t)}{X(\omega,t)} = \mu i(\omega,t),$$

where μ is a positive constant.

Whenever a firm succeeds in innovating, it acquires the uncertain stream of profit flows that accrues to a monopolist, that is, the stock market valuation of the firm, $v(\omega,t)$. Thus, the problem faced by an R&D firm is that of choosing the amount of labor input in order to maximize its expected profits[3]

$$\max_{l_k} \left[\frac{v(\omega,t)A(\omega)}{X(\omega,t)} l_k - l_k \right].$$

The problem above provides a finite, positive solution for l_k only when the arbitrage equation

$$\frac{v(\omega,t)A(\omega)}{X(\omega,t)} = 1$$

is satisfied. Efficient financial markets require that the stock market valuation of the firm yields an expected rate of return equal to the riskless interest rate $r(t)$. The firm's market valuation is

$$v(\omega,t) = \frac{\pi(\omega,t)}{r(t) + \frac{A(\omega)L_I(\omega,t)}{X(\omega,t)} - \frac{\dot{v}(\omega,t)}{v(\omega,t)}},$$

that is, the present value of profits discounted at the obsolescence-adjusted interest rate (see Grossman and Helpman, 1991). Finally, the R&D equilibrium condition is

$$\frac{\pi(\omega,t)A(\omega)}{X(\omega,t)\left[r(t) + \frac{A(\omega)L_I(\omega,t)}{X(\omega,t)} - \frac{\dot{v}(\omega,t)}{v(\omega,t)}\right]} = 1. \tag{4}$$

Since in each industry the market demands, $D(\omega,t) = \left[\alpha(\omega)E(t)L(t)\right]/\lambda(\omega)l_\omega$, require $D(\omega,t)l_\omega$ units of labor in order to be produced, the total employment in the manufacturing sector is

$$\int_0^1 \frac{\alpha(\omega)E(t)L(t)}{\lambda(\omega)} d\omega.$$

[2] The acronym TEG stands for 'Temporary effects on growth' of policy measures such as subsidies and taxes. Useful surveys on the scale effect problem and the way it has been solved are Dinopoulos and Thompson (1999) and Jones (1999).

[3] We consider labor as numerarie and normalize the wage rate to 1.

As a result, the labor market-clearing condition implies

$$L(t) = \int_0^1 \frac{\alpha(\omega)E(t)L(t)}{\lambda(\omega)} d\omega + \int_0^1 L_I(\omega, t) d\omega. \tag{5}$$

where $\int_0^1 L_I(\omega, t)d\omega$ is the total employment in the research sector.

We now focus on the steady state equilibrium, where all variables grow at constant rates. Along the steady state $\dot{E}(t)/E(t) = 0$ and hence, from the Euler equation, $r(t) = \rho$. Moreover, from the definition of $v(\omega, t)$ it follows that its steady state growth rate is $\dot{v}(\omega, t)/v(\omega, t) = n$. By solving the system made up of (4) and (5), we obtain the steady-state values of expenditure E^*, and of current and expected R&D efforts $L_I^*(\omega, t)$ - which coincide in the rational expectations equilibrium - as

$$E^* = \frac{\frac{\mu}{n}\rho + 1 - \mu}{\left(\frac{\mu}{n}\rho - \mu\right)\int_0^1 \frac{\alpha(\omega)}{\lambda(\omega)} d\omega + 1}$$

and

$$L_I^*(\omega, t) = L(t) \frac{\alpha(\omega) - \frac{\alpha(\omega)}{\lambda(\omega)}}{\left(\frac{\mu}{n}\rho - \mu\right)\int_0^1 \frac{\alpha(\omega)}{\lambda(\omega)} d\omega + 1}. \tag{6}$$

Notice that the steady state research investments are industry-specific and that, by construction, they equalize R&D returns across industries.

3 The Robustness of the Return-Equalizing Equilibrium

The equalization of R&D returns leaves the investor indifferent as to how to allocate resources across industries. As argued in the Introduction, this indifference justifies the assumption we make in this section, that is, the investors' aversion against uncertainty. We characterize the agents' R&D investment strategy, and we show that the return-equalizing equilibrium is robust against uncertainty aversion.

Importantly, our assumption on the agents' attitude towards uncertainty does not concern any fundamental of the economy and is to be interpreted as a way of treating the *extrinsic uncertainty* (Cass and Shell, 1983) associated to the future configuration of R&D investments across industries.[4]

[4] Moreover, uncertainty does not affect expectations on the *aggregate* amount of research. Uncer-

Assume that the investor is $(1-p)100\%$ sure to face in the future the return-equalizing configuration of R&D investment and that, with a however small probability p, any other possible configuration can occur. We can call this situation a "$p-$contamination of confidence".[5] Aversion to ambiguity in this context implies that with probability p the agent expects the worst configuration of future R&D investment, that is, the one which minimizes her expected returns.[6] Since the minimizing configuration is a function of the agent's investment choice, the decision problem can be formalized as a "two-player zero-sum game" in which (i) a "malevolent Nature" selects the worst possible configuration of *future* R&D efforts while (ii) the agent selects the best possible configuration of *current* R&D efforts.

We denote with $l_m(t)+\gamma(\omega,t)$ the agent's investment in industry ω at time t, and with $L_I^e(t)+\varepsilon(\omega,t)$ the aggregate expected research in industry ω, at time t. l_m and L_I^e are, respectively, the agent's average investment per industry and the average expected research per industry. $\varepsilon(\cdot)$ and $\gamma(\cdot)$ represent deviations from the averages satisfying

$$\int_0^1 \varepsilon(\omega,t)d\omega=0;\quad \int_0^1 \gamma(\omega,t)d\omega=0;\quad \varepsilon(\omega,t)>-L_I^e(t);\quad \gamma(\omega,t)>-l_m(t).$$

The presence of the two functions $\gamma(\cdot)$ and $\varepsilon(\cdot)$ is intended to allow for asymmetry across industries both in the agent's investment and in expected research.

We can now state the R&D investment problem as

$$\max_{\gamma(.)}\left[\min_{\varepsilon(.)}\int_0^1 [l_m(t)+\gamma(\omega,t)]\left(p\tfrac{A(\omega)v(\omega,t)}{X(\omega)}+(1-p)q(t)\right)d\omega\right]$$

$$\text{s.t. (i), (ii)}\ \int_0^1 \gamma(\omega,t)d\omega=\int_0^1 \varepsilon(\omega,t)d\omega=0;\ \text{(iii)}\ \varepsilon(\omega,t)>-L_I^e(t);\ \text{(iv)}\ \gamma(\omega,t)>-l_m(t)$$

tainty arises from the fact that agents are indifferent among a set of choices. This is not the case for the total amount of research: if agents expect the equilibrium aggregate amount of research, their choice between consumption and savings, which are channelled to the research sector, is uniquely determined and confirms their expectations; there is no indifference and no uncertainty.

[5] In the literature this situation is usually referred to as $\varepsilon-$contamination (which is also how we called it in the Introduction). However, as we will see, in our context ε stands for the extension of the state space.

[6] See the representation theorem (theorem 1) in Nishimura and Ozaki (2006) for an axiomatization of the choice behavior assumed here.

where

$$v(\omega,t) \equiv \frac{\pi(\omega,t)}{r(t)+(1-\mu)\,\frac{A(\omega)}{X(\omega,t)}\left[L_I^e(t)+\varepsilon(\omega,t)\right]},$$

and where $q(t)$ is defined as the expected R&D return which, with probability $1-p$, is constant across industries.

In Appendix A we solve the maxmin problem above via the calculus of variations and characterize the agent's investment strategy as

$$l_m(t)+\gamma(\omega,t) = l_m(t)\frac{\pi(\omega,t)}{\int_0^1 \pi(\omega,t)d\omega} \qquad \forall\omega\in[0,1], \tag{7}$$

and the distribution of expected R&D investments as

$$L_I^e(t)+\varepsilon(\omega,t) = \frac{\pi(\omega,t)}{\int_0^1 \pi(\omega,t)d\omega}\left(L_I^e(t)+\frac{r(t)}{(1-\mu)}\int_0^1 \frac{X(\omega,t)}{A(\omega)}d\omega\right)-\frac{r(t)}{(1-\mu)}\frac{X(\omega,t)}{A(\omega)} \quad \forall\omega\in[0,1]. \tag{8}$$

We can now turn to the steady state and enunciate the following

Proposition 1 *For a however small probability (p) of deviation ($\varepsilon(\omega)$) from the return-equalizing expectations on the future R&D investment, decision makers adopting a maxmin strategy to solve their investment allocation problem choose a steady state investment strategy which equalizes R&D returns across industries. The values of these investments coincide with those in (6).*

Proof. See Appendix B. ■

Intuitively, the contamination of confidence implies that, whenever the agent evaluates an investment allocation different from the return-equalizing one, she will always be induced to expect the worst configuration of future investments inside the ε-generated set. As a result, even though the agent is 'almost sure' ($p \to 0$) to face in the future the return-equalizing configuration of R&D investment (which would leave her in a position of indifference in her current allocation problem), the mere possibility of a different future configuration (as captured by $\varepsilon(\omega)$) makes her strictly prefer to choose the return-equalizing R&D investment strategy. This strategy then arises as the unique optimal investment allocation.

4 Concluding Remarks

The fact that R&D investment decisions are taken under conditions of severe uncertainty about their returns has long been recognized in the economics literature (see among others Rosenberg (1994) and Freeman and Soete (1997)): innovations are "unique" events, and the process aimed at producing them is an uncertain and largely unpredictable economic activity. The concept of "Knightian uncertainty" (as opposed to "risk") may prove helpful in the analysis of the evolution of the innovation process in modern economies. Recent studies on ambiguity (and ambiguity attitude) have tried to give an "operational" meaning to Knightian uncertainty. We have adopted the multiple-prior approach pioneered by Gilboa and Schmeidler (1989). In particular, in a vertical innovation growth model with asymmetric fundamentals we have explored the relationship between ambiguity and "extrinsic uncertainty", that is, uncertainty not related to the economy's fundamentals but lying in the current evaluation of R&D investments to be carried out by future investors. We have shown that a however small degree of ambiguity aversion eliminates the indeterminacy in the R&D investment allocation problem. As a result, the family of vertical innovation models can be meaningfully extended to more realistic asymmetric frameworks where the return-equalizing equilibrium is univocally identified as the unique rational expectations equilibrium.

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A The Maxmin Problem

$$\max_{\gamma(\cdot)}\left[\min_{\varepsilon(\cdot)}\int_0^1\left[l_m(t)+\gamma(\omega,t)\right]v(\omega,t)\frac{A(\omega)}{X(\omega,t)}d\omega\right]$$

$$\text{s.t.}\quad\int_0^1\gamma(\omega,t)d\omega=\int_0^1\varepsilon(\omega,t)d\omega=0;\quad\varepsilon(\omega,t)>-L_I^e(t);\quad\gamma(\omega,t)>-l_m(t)$$

where

$$v(\omega,t)=\frac{\pi(\omega,t)}{r(t)-\frac{\dot{v}(\omega,t)}{v(\omega,t)}+\frac{A(\omega)}{X(\omega,t)}\left[L_I^e(t)+\varepsilon(\omega,t)\right]}.$$

Under TEG specification

$$\frac{\dot{X}(\omega,t)}{X(\omega,t)}=\mu\frac{A(\omega)}{X(\omega,t)}\left[L_I^e(t)+\varepsilon(\omega,t)\right].$$

Moreover, as by differentiating (4) with respect to time, we obtain $\dot{v}(\omega,t)/v(\omega,t)=\dot{X}(\omega,t)/X(\omega,t)$, then

$$v(\omega,t)=\frac{\pi(\omega,t)}{r(t)+(1-\mu)\frac{A(\omega)}{X(\omega,t)}\left[L_I^e(t)+\varepsilon(\omega,t)\right]}.$$

From the definition of probability p the return from any investment is industry specific $(v(\omega,t)A(\omega)/X(\omega,t))$ with probability p, while it is constant across industries with probability $(1-p)$ (let us define this constant value as $q(t)$). Then the problem is equivalent to

$$\max_{\gamma(.)}\left[\min_{\varepsilon(.)}\int_0^1[l_m(t)+\gamma(\omega,t)]\left(p\frac{A(\omega)\pi(\omega,t)}{X(\omega)\left(r(t)+(1-\mu)\frac{A(\omega)}{X(\omega,t)}\left[L_I^e(t)+\varepsilon(\omega,t)\right]\right)}+(1-p)q(t)\right)d\omega\right]=$$

$$=\quad(1-p)q(t)+p\max_{\gamma(.)}\left[\min_{\varepsilon(.)}\int_0^1[l_m(t)+\gamma(\omega,t)]\frac{A(\omega)\pi(\omega,t)}{X(\omega)\left(r(t)+(1-\mu)\frac{A(\omega)}{X(\omega,t)}\left[L_I^e(t)+\varepsilon(\omega,t)\right]\right)}d\omega\right],$$

which admits the same solution as

$$\max_{\gamma(.)}\left[\min_{\varepsilon(.)}\int_0^1[l_m(t)+\gamma(\omega,t)]\frac{A(\omega)\pi(\omega,t)}{X(\omega)\left(r(t)+(1-\mu)\frac{A(\omega)}{X(\omega,t)}\left[L_I^e(t)+\varepsilon(\omega,t)\right]\right)}d\omega\right].$$

Notice that this is valid for a however small probability p. Given these conditions, we first solve for the minimization problem

$$\min_{\varepsilon(\cdot)}\int_0^1\frac{[l_m(t)+\gamma(\omega,t)]\,\pi(\omega,t)}{\frac{X(\omega,t)}{A(\omega)}r(t)+(1-\mu)(L_I^e(t)+\varepsilon(\omega,t))}d\omega$$

$$\text{s.t.}\quad\int_0^1\varepsilon(\omega,t)d\omega=0.$$

We set $e(\omega,t)=\int_0^\omega\varepsilon(s,t)ds$; then $e'(\omega,t)=\varepsilon(\omega,t)$ $\forall\omega\in[0,1]$ and the minimization problem ($\text{P}_{\min}$) can be expressed as

$$\min_{e'(\cdot)}\int_0^1 G(e')d\omega$$

$$\text{s.t.}\;\; e(0)=0;\quad e(1)=0$$

where

$$G(e')=\frac{[l_m(t)+\gamma(\omega,t)]\,\pi(\omega,t)}{\frac{X(\omega,t)}{A(\omega)}r(t)+(1-\mu)(L_I^e(t)+\varepsilon(\omega,t))}.$$

This is the simplest problem of calculus of variations. Since under the conditions specified above $G(e')\in C^2$, we can apply the Euler theorem stating that, if $G(e,e',\omega)\in C^2$ and e^* is optimal and C^1, then e^* must necessarily solve

$$G_e-\frac{d}{d\omega}G_{e'}=0. \tag{9}$$

As in our case G does not depend on e, $G_e = 0$, and hence (9) becomes $(d/d\omega)\, G_{e'} = 0$, implying that

$$G_{e'} \equiv G_{\varepsilon} = -\frac{\pi(\omega,t)[l_m(t)+\gamma(\omega,t)]}{\left[\frac{X(\omega,t)}{A(\omega)}r(t)+(1-\mu)(L_I^e(t)+\varepsilon(\omega,t))\right]^2}$$

be constant with respect to ω. Hence

$$\frac{\pi(\omega,t)\,[l_m(t)+\gamma(\omega,t)]}{\left[\frac{X(\omega,t)}{A(\omega)}r(t)+(1-\mu)(L_I^e(t)+\varepsilon(\omega,t))\right]^2} = k_1,$$

where k_1 is a real constant. Now we solve the expression above for $\varepsilon(\omega,t)$ and obtain the *reaction function* of "Nature" to the agent's decision as

$$\varepsilon(\omega,t) = \sqrt{\frac{\pi(\omega,t)\,[l_m(t)+\gamma(\omega,t)]}{k_1(1-\mu)}} - \frac{X(\omega,t)}{A(\omega)(1-\mu)}r(t) - L_I^e(t). \tag{10}$$

We can now plug it into the maximization problem ($P_{\max}$) and solve for γ:

$$\max_{\gamma(.)} \int_0^1 [l_m(t)+\gamma(\omega,t)] \frac{\pi(\omega,t)}{\sqrt{\frac{\pi(\omega,t)\,[l_m(t)+\gamma(\omega,t)]\,(1-\mu)}{k_1}}} d\omega$$

$$\text{sub} \int_0^1 \gamma(\omega,t)d\omega = 0.$$

Rearranging, this problem becomes

$$\max_{\gamma(.)} \int_0^1 [l_m(t)+\gamma(\omega,t)]^{\frac{1}{2}} \left(\pi(\omega,t)k_1/(1-\mu)\right)^{\frac{1}{2}} d\omega$$

$$\text{sub} \int_0^1 \gamma(\omega,t)d\omega = 0.$$

Again, we solve $P_{\max}$ as a problem of calculus of variations. By setting $c(\omega,t) = \int_0^{\omega} \gamma(s,t)ds$, so that $c'(\omega,t) = \gamma(\omega,t)$, $P_{\max}$ becomes

$$\max_{c'} \int_0^1 F(c')d\omega$$

$$\text{sub} \quad c(0) = 0; c(1) = 0$$

where $F(c') \equiv F(\gamma) = [l_m(t)+\gamma(\omega,t)]^{\frac{1}{2}} [\pi(\omega,t)k_1]^{\frac{1}{2}}$. With the same reasoning as before, the Euler theorem, $F_c - \frac{d}{d\omega}F_{c'} = 0$, implies

$$F_{c'} \equiv F_\gamma = -\frac{(\pi(\omega,t)k_1)^{\frac{1}{2}}}{2[l_m + \gamma(\omega,t)]^{\frac{1}{2}}} = -k_2,$$

where $k_2 \in R_+$. From F_γ we can derive the expression for $\gamma(\omega,t)$ as

$$\gamma(\omega,t) = \frac{\pi(\omega,t)k_1}{4k_2^2} - l_m. \tag{11}$$

Plugging it into (10), we obtain

$$\begin{aligned} \varepsilon(\omega,t) &= \sqrt{\frac{\pi(\omega,t)\left[l_m(t) + \frac{\pi(\omega,t)k_1}{4k_2^2} - l_m(t)\right]}{k_1}} - \frac{X(\omega,t)}{A(\omega)(1-\mu)}r(t) - L_I^e(t) = \quad (12) \\ &= \frac{\pi(\omega,t)}{2k_2} - \frac{X(\omega,t)}{A(\omega)(1-\mu)}r(t) - L_I^e(t). \end{aligned}$$

Now we can use the two conditions imposed by the constraints

$$\int_0^1 \gamma(\omega,t)d\omega = 0 \iff \int_0^1 \left[\frac{\pi(\omega,t)k_1}{4k_2^2} - l_m(t)\right] d\omega = 0,$$

$$\int_0^1 \varepsilon(\omega,t)d\omega = 0 \iff \int_0^1 \left[\frac{\pi(\omega,t)}{2(1-\mu)k_2} - \frac{X(\omega,t)}{A(\omega)(1-\mu)}r(t) - L_I^e(t)\right] d\omega = 0,$$

to find constants

$$k_1 = \frac{4k_2^2(1-\mu)l_m(t)}{\int_0^1 \pi(\omega,t)d\omega} \tag{13}$$

and

$$k_2 = \frac{\int_0^1 \pi(\omega,t)d\omega}{2(1-\mu)\left[\frac{r(t)}{(1-\mu)}\int_0^1 \frac{X(\omega,t)}{A(\omega)}d\omega + L_I^e(t)\right]}. \tag{14}$$

Substituting (14) into (13), we obtain

$$k_1 = \frac{l_m(t)\int_0^1 \pi(\omega,t)d\omega}{(1-\mu)\left[\frac{r(t)}{(1-\mu)}\int_0^1 \frac{X(\omega,t)}{A(\omega)}d\omega + L_I^e(t)\right]^2}. \tag{15}$$

Finally we can plug (14) and (15) into (11) and (12) in order to obtain the optimal pair $\gamma^*(\omega,t)$, $\varepsilon^*(\omega,t)$ as

$$\gamma^*(\omega,t) = \frac{\pi(\omega,t)k_1}{4k_2^2} - l_m(t) = l_m(t)\left[\frac{\pi(\omega,t)}{\int_0^1 \pi(\omega,t)d\omega} - 1\right]$$

and

$$\begin{aligned}
\varepsilon^*(\omega,t) &= \frac{\pi(\omega,t)}{\int\limits_0^1 \pi(\omega,t)d\omega}\left[\frac{r(t)}{(1-\mu)}\int\limits_0^1 \frac{X(\omega,t)}{A(\omega)}d\omega + L_I^e(t)\right] - \frac{r(t)}{(1-\mu)}\frac{X(\omega,t)}{A(\omega)} - L_I^e(t) = \\
&= L_I^e(t)\left[\frac{\pi(\omega,t)}{\int\limits_0^1 \pi(\omega,t)d\omega} - 1\right] + \frac{\pi(\omega,t)}{\int\limits_0^1 \pi(\omega,t)d\omega}\frac{r(t)}{(1-\mu)}\int\limits_0^1 \frac{X(\omega,t)}{A(\omega)}d\omega - \frac{r(t)}{(1-\mu)}\frac{X(\omega,t)}{A(\omega)},
\end{aligned}$$

from which we can easily obtain expressions (7) and (8).

B Characterization of the Steady State and Proof of Proposition 1

Expressions (7) and (8) prove to be relevant as soon as we turn to the *steady-state* equilibrium. Then

$$\frac{\dot{X}(\omega,t)}{X(\omega,t)} \equiv \mu i(\omega) = n$$

and, as $\dot{E}(t)/E(t) = 0$, it is $r(t) = \rho$. It is easy to show, by substituting for $L_I^e(t) + \varepsilon(\omega,t)$ (as given in (8)) into $v(\omega,t)$, that the R&D returns $(v(\omega,t)A(\omega)/X(\omega))$ are equalized across industries. In fact

$$\begin{aligned}
\frac{A(\omega)v(\omega)}{X(\omega)} &= \frac{\pi(\omega,t)A(\omega)}{X(\omega,t)\rho + \frac{A(\omega)L_I^e(t)+\varepsilon(\omega,t)}{X(\omega,t)}(1-\mu)} = \\
&= \frac{\pi(\omega,t)}{\frac{X(\omega,t)}{A(\omega)}\rho + (1-\mu)\left(\frac{\pi(\omega,t)}{\int\limits_0^1 \pi(\omega,t)d\omega}\left(L_I^e(t) + \frac{\rho}{(1-\mu)}\int\limits_0^1 \frac{X(\omega,t)}{A(\omega)}d\omega\right) - \frac{\rho}{(1-\mu)}\frac{X(\omega,t)}{A(\omega)}\right)} = \\
&= \frac{\pi(\omega,t)}{\frac{\mu}{n}\rho\left(L_I^e(t)+\varepsilon(\omega,t)\right) + (1-\mu)\left(\frac{\pi(\omega,t)}{\int\limits_0^1 \pi(\omega,t)d\omega}\left(L_I^e(t) + \frac{\rho}{(1-\mu)}\frac{\mu}{n}L_I^e(t)\right) - \frac{\rho}{(1-\mu)}\frac{\mu}{n}\left(L_I^e(t)+\varepsilon(\omega,t)\right)\right)}.
\end{aligned}$$

Substituting for the steady state value of $X(\omega,t)=\dfrac{\mu}{n}A\left(L_I^e(t)+\varepsilon(\omega,t)\right))$, we finally obtain

$$\begin{aligned}\frac{A(\omega)v(\omega)}{X(\omega)} &= \frac{\pi(\omega,t)}{(1-\mu)\dfrac{\pi(\omega,t)}{\int_0^1 \pi(\omega,t)d\omega}L_I^e(t)+\dfrac{\rho\mu}{n}\dfrac{\pi(\omega,t)}{\int_0^1 \pi(\omega,t)d\omega}L_I^e(t)} = \\ &= \left(\frac{L_I^e(t)}{EL(t)}\left(1-\mu+\frac{\rho\mu}{n}\right)\right)^{-1}.\end{aligned}$$

Now, by using the arbitrage equation for any industry (equation (4)), we can solve for $L_I^e(t)$ and obtain

$$L_I^e(t)=\frac{EL(t)\left(1-\frac{\alpha(\omega)}{\lambda(\omega)}\right)}{\frac{\mu}{n}\rho+1-\mu},$$

or, in per capita terms,

$$l_m^e=\frac{E\left(1-\frac{\alpha(\omega)}{\lambda(\omega)}\right)}{\frac{\mu}{n}\rho+1-\mu}. \tag{16}$$

Dividing the market-clearing condition

$$L(t)=\int_0^1 \frac{\alpha(\omega)EL(t)}{\lambda(\omega)}d\omega+L(t)\int_0^1\left[l_m+\gamma(\omega,t)\right]d\omega.$$

by $L(t)$, we can write

$$1=E\int_0^1\frac{\alpha(\omega)}{\lambda(\omega)}d\omega+l_m. \tag{17}$$

Given the absence of uncertainty on aggregate, and average, expected amount of research, then $l_m^e(t)=l_m(t)$. The steady-state resource (17) and arbitrage (16) equations allow us to find the equilibrium values of l_m and E as

$$E=\frac{\frac{\mu}{n}\rho+1-\mu}{\left(\frac{\mu}{n}\rho-\mu\right)\int_0^1\frac{\alpha(\omega)}{\lambda(\omega)}d\omega+1}$$

and

$$l_m=\frac{1-\int_0^1\frac{\alpha(\omega)}{\lambda(\omega)}d\omega}{\left(\frac{\mu}{n}\rho-\mu\right)\int_0^1\frac{\alpha(\omega)}{\lambda(\omega)}d\omega+1}.$$

The proof of proposition 1 is now straightforward. By plugging the mean value l_m into expression (7) we obtain

$$\begin{aligned} l_m + \gamma(\omega, t) &= l_m \frac{\pi(\omega, t)}{\int_0^1 \pi(\omega, t) d\omega} = \frac{1 - \int_0^1 \frac{\alpha(\omega)}{\lambda(\omega)} d\omega}{\left(\frac{\mu}{n}\rho - \mu\right) \int_0^1 \frac{\alpha(\omega)}{\lambda(\omega)} d\omega + 1} \cdot \frac{\alpha(\omega) - \frac{\alpha(\omega)}{\lambda(\omega)}}{1 - \int_0^1 \frac{\alpha(\omega)}{\lambda(\omega)} d\omega} = \\ &= \frac{\alpha(\omega) - \frac{\alpha(\omega)}{\lambda(\omega)}}{\left(\frac{\mu}{n}\rho - \mu\right) \int_0^1 \frac{\alpha(\omega)}{\lambda(\omega)} d\omega + 1}. \end{aligned}$$

Since by definition $L_I^*(\omega, t) \equiv L(t) \left[l_m + \gamma(\omega, t) \right]$, steady state R&D investments coincide with those given in (6).