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Centralized Wage Setting and Labor Market Policies: the Nordic Model Case*

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Abstract

It is often argued that rigid labour market and centralized bargaining are harmful employment and growth. This paper looks at the case of Nordic countries as a counter-example pointing to some weaknesses of this view. Rigid labour markets, while reducing the offer of low quality jobs, increase average labor productivity by favoring job relocation in high quality jobs. Moene and Wallerstein (1997) adopted a vintage-capital model to compare centralized and decentralized bargaining: they show that centralized bargaining systems yield higher labor productivity and higher structural unemployment. By introducing a frictional labor market in the vintage-capital framework, we show that the negative effects on employment characterizing centralized bargaining can be reduced by adopting active labor market policy.

Keywords: Centralized wage setting, structural change, labor market policy, frictional unemployment

JEL Classification: J31, J60, L16.

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1 Introduction

The ‘Nordic’ model provides a way to look at the relation between inequality, productivity and employment compatible with the social democratic goals of combining egalitarian distribution of earnings, security of income and efficiency (see Moene 2008). The original formulation of the model is due to two Swedish trade union economists - Gosta Rehn and Rudolf Meidner- and dates back to the 40s. Later on, Rehn and Meidner perfected it and advocated its implementation by the Swedish government throughout the 50s and 60s.

Three main policies constituted the core of the model: restrictive fiscal policy, active labor market policies and wage policy of solidarity. In the context of a small open economy during the post World War II boom, fighting inflation was a bigger concern than stimulating aggregate demand, which was kept high by the external channel: fiscal restraint served this purpose. The two other policies together aimed at fostering structural change while guaranteeing distributive equality and high employment. Centralized bargaining was at the centre of the system. By negotiating equal remuneration for identical jobs (‘Equal pay for equal jobs’) regardless of the productivity of plants or firms, centralized wage bargaining was thought of as a tool capable not only of providing the equalization of earnings but also of fostering productivity growth. A more compressed distribution of wages would put pressure on low productive plant and it would oblige them either to rationalize production, thus increasing productivity directly, or to shut down, thus freeing resources potentially employable by more dynamic and productive firms or sectors. At the same time, wage compression would act as a subsidy to investment in more productive plants by increasing their relative value, and therefore enhancing the scope for job relocation in high-tech activities. Labor market policies complement wage solidarity as they help the transition of labor from low to high productive firms, sectors or regions. Labor market policies could be either universal (matching policies and employment subsidies); or selective (supply-side retraining, vocational education, relocation grants).

It is a matter of debate whether this model has been faithfully implemented (see Erixon 2008). Centralized wage-setting, however, became a distinctive feature of Swedish economic policy between 1956 and 1983. Two different phases can be distinguished. Phase I began in 1956, when national unions of blue-collar workers (LO) and employers (SAF) found the first com-

prehensive framework agreement for private blue-collar workers; it lasted till the late 60s. During this period solidarity wage policy was properly applied according to the principle ‘equal pay for equal jobs’, and centralized agreements favored wage equalization among analogous jobs in different industries and plants (Hibbs and Locking 2000, p. 760). In phase II, which began in the early 70s and ended in 1983 when the last comprehensive agreement was signed, the main goal of wage solidarity shifted from facilitating structural change to achieving wage equalization *per se*, irrespective of the type of job. As a result, wage inequality was reduced not only across plants and industries, but also within plants and across skills grade. In fact, during the 70s both returns to and investment in human capital drastically declined, especially for university education, thus possibly contributing to a reduction in productivity growth (Leamer and Lundborg 1997, Lindbeck 1997). Empirical evidence on the relation between wage dispersion and productivity (Hibbs and Locking 2000) supports the view that a reduction in ‘across plants’ wage inequality positively affects labor productivity growth, while a reduction in ‘within plants’ wage inequality—accompanied by an equalization across skill levels—would be harmful. Such difficulties led to a progressive abandonment of centralized national bargaining, which after 1983 mostly took place at industry and firms level. Wage inequality regained ground, but currently it still stands at levels substantially lower than in Anglo-American economies (see Pontusson 2006).

A possible formalization of the Rehn-Meidner model has been provided by Moene and Wallerstein (1997, MW henceforth). They compare the performances of centralized and decentralized wage bargaining in terms of productivity and employment outcomes. They do not, however, take into explicit consideration the role of labor market policies as originally suggested by Rehn and Meidner. We adopt the framework proposed by Moene and Wallerstein and we extend their analysis to consider the relevance of labor market policies. To the purpose, we will consider a vintage model with frictional unemployment along the lines of Hornstein, Krusell and Violante (2008, HKV henceforth).

2 Reminder of the Moene and Wallerstein model

In this section we provide a simplified reminder of the model developed by MW (1997). Technical progress occurs at the exogenous rate $\gamma > 0$, and it is embodied in new plants. Since investment costs are sunk there is a distribution of plants of different ages: plants will be active until they are productive enough to pay (labor) variable costs. Labor per plant ratio is fixed and normalized to one. Accordingly, value added per worker of plant of age t can be represented by

$$y(t) = y_0 e^{\gamma t}. \quad (1)$$

Relative price of investment (I) to output is assumed constant over time, so that investment cost follows the same exponential path as value added:

$$C(I(t)) = C(I(0)) e^{\gamma t}. \quad (2)$$

Profits per worker at time s earned on a plant of age t are given by

$$\pi(s, t) = y(t) - w(s, t), \quad (3)$$

where $w(s, t)$ is the wage paid on plant of age t at time s . Market value of a new plant is given by the discounted value of future profits earned on the plant:

$$V(t) = \int_t^{t+\vartheta} e^{-\rho(s-t)} \pi(s, t) ds, \quad (4)$$

where ρ is the discount rate and ϑ is the expected lifetime of a plant. Free entry ensures that investment will be pushed to the level where cost and market value are equal, so that:

$$V(t) = C(I(t)). \quad (5)$$

Since plants will be active until they yield non negative profits, the age of the oldest plant in operation at time s can be obtained by solving:

$$\pi(s, s - \vartheta) = y(s - \vartheta) - w(s, s - \vartheta) = 0. \quad (6)$$

Finally, labor supply (normalized to one) constrains investment given the fixed capital-labor ratio:

$$\int_{s-\vartheta}^s I(t)dt \leq 1. \quad (7)$$

Focusing on steady states where wages and labor productivity grow at the same rate γ , equations (5), (6), and (7) determine ϑ , I , and the market clearing wage rate $r(s)$. Notice that ϑ can be interpreted as an (inverse) measure of productivity: the longer the optimal lifetime of a plant, the lower the average productivity.

Within this framework, MW compare the effects of local and centralized bargaining by adopting two different rules for wage formation. Under local bargaining, workers can appropriate a share (a) of their vintage-specific productivity $y(t)$, so that

$$w(s, t) = \max(ay(t), r(s)), \quad (8)$$

Under centralized bargaining, workers earn a share of the average productivity ($\bar{y}(s) \equiv (1/\vartheta) \int_{s-\vartheta}^s y(t)dt$):

$$w(s) = \max(a\bar{y}(s), r(s)), \quad (9)$$

In their analysis, MW show that centralized bargaining is always superior to local bargaining in terms of steady state productivity while always inferior in terms of employment; if $a\bar{y}(t) > r(s)$ (the relevant case), centrally bargained wage is too high to employ the entire labor force, while in the local bargaining case there is full employment by assumption. The effects on investment and total output (the product between investment and labor productivity) will depend on a . At low levels of the workers' share of productivity the centralized system turns out to be superior, whereas for a higher than a certain threshold value local bargaining performs better. This is consistent with the Swedish experience: in phase II, the observed slowdown in private-sector employment has been the response to an increase in unions' wage claims (Lindbeck 1997).

The rationale underlying these results is that, in order to speed up the pace of adoption of new plants, centralized wage setting needs to put pressure on low productive vintage by raising the wage rate above its competitive, full employment, level. The outcome is a trade-off between productivity and employment, with the final effect on total production depending on a .

The model provides a faithful representation of the way solidarity wage policy was conceived. However, it does not consider the role of labor market and retraining policies, thus neglecting the other main pillar of the original view of the Rehn-Meidner model. In the next section we show how the introduction of a frictional labor market in the MW framework allows gaining a scope for active labor market policies.¹ As it turns out, matching policies reduce unemployment associated to the centralized wage setting not only by reducing unemployment duration, but also by increasing employment growth thanks to a positive effect on the expected value of investment.

3 The model with frictional labor market

Our analysis of capital-embodied productivity growth in the context of a frictional labor market follows the standard search-vintage approach (Aghion and Howitt (1994), Mortensen and Pissarides (1998)), recently enriched by HKV (2008). In particular, as in HKV, firms purchase capital before being matched to a worker; only after sustaining the investment sunk cost they engage in the search of workers in the frictional labor market.

3.1 Labor market

Our main point of departure from the standard analysis consists in abandoning the simultaneous determination of labor market tightness and scrapping age through the job destruction and job creation equations. When centralized bargaining is assumed, the exit condition (6) determines the scrapping age of plant ϑ as the solution to:

$$y(s - \vartheta) - a\bar{y}(s) = 0, \tag{10}$$

which is implicitly given by $a = \gamma\vartheta/(e^{\gamma\vartheta} - 1)$.

Once the age of the oldest active plant is known, our framework will be able to solve for equilibrium unemployment and the level of investment.

¹Note that here frictions are an implicit measure of the degree of heterogeneity in the labor market. This view is consistent with the modern view of labour market frictions given by Pissarides (2000). In particular, with respect to the old view that consider frictional unemployment as the outcome of labour supply decisions, frictions emerge in a world where workers and firms are heterogeneous and have imperfect information about the characteristics of their counterpart.

Plants are productive only when matched to a worker. The meeting process between firms and workers is governed by the constant returns to scale matching function $m(v, u)$ which gives the number of matches per unit of time as a function of the number of vacancies (v) and unemployed workers (u). A firm meets a worker with probability $\lambda_f = m(v, u)/v$. A worker meets a firm with probability $\lambda_w = m(v, u)/u$, hence $1/\lambda_w$ is the mean duration of unemployment. Let $\zeta \equiv v/u$, then $\lambda_f = m(\zeta, 1)/\zeta$, and $\lambda_w = m(\zeta, 1)$. On the other hand, existing matches dissolve at the exogenous rate σ .

Steady state equilibrium requires the equality between the flow into and the flow out of unemployment. Let $\mu(t)$ be the measure of employment on vintage t , so that total employment be $\mu = \int_{s-\vartheta}^s \mu(t)dt$. Since at age $t = \vartheta$ plants are scrapped, the flow into unemployment is equal to $\sigma\mu + \mu(\vartheta)$; while the flow out of unemployment is $m(\zeta, 1)u$. Steady state unemployment, in turn, satisfies:

$$\frac{u}{1-u} = \frac{1}{m(\zeta, 1)} \left(\sigma + \frac{\mu(\vartheta)}{\mu} \right) \quad (11)$$

where we used $\mu = 1 - u$; and where $\mu(\vartheta)/\mu = [1 - e^{-(\sigma+\lambda_f)\vartheta}] / \left[\vartheta - \frac{1}{\sigma+\lambda_f}(1 - e^{-(\sigma+\lambda_f)\vartheta}) \right]$ (see HKV, p.1099).

3.2 Equilibrium

Taking into account frictions in the labor market and the possibility that plants be idle, free entry condition (5) need be modified as:

$$(m(\zeta, 1)/\zeta)V(t) = C(I(t)). \quad (5 \text{ bis})$$

The system made up of (10), (11) and (5 bis) determines steady state productivity (ϑ), unemployment, and investment. Notice that we have dropped labor market tightness (ζ) from the endogenous of the model as it is determined as a function of u and I : $\zeta \equiv v/u = (\vartheta I - (1 - u))/u$. The number of vacancies is equal to the number of existing jobs (ϑI) minus employment. The assumption of a uniform wage set at a level higher than the competitive one leads to structural unemployment as the incentive to invest is not strong enough to yield a number of plants capable of employing the whole labor force: $\vartheta I - 1 < 0$. Were frictions absent from the labor market ($v = 0$), we would have $u = 1 - \vartheta I$, so that total and structural unemployment would coincide. On the contrary, a frictional labor market ($v > 0$),

implies $u > (1 - \vartheta I)$: total unemployment consists of the sum of structural and frictional unemployment.

4 Policy

In a vintage model with frictions, labor market policies can be seen as policies that favor the relocation of labor from old to new plants. In this context, the joint effect of centralized wage bargaining and labor market policies might enhance workers' relocation towards more productive plants.

In order to investigate this issue, we introduce labor market policies into the model and we analyze their effect on the steady state values of investment, employment and productivity. We focus on matching policies, i.e. all policies which make the meeting between a vacant firm and an unemployed worker easier. We represent such policies by means of a shift variable in the matching function. Let z be expenses in labor market policies, the matching function becomes $m(v, u, z)$, with $m_z > 0$. In particular, in accordance with several empirical studies (see Pissarides 2000), let us assume a Cobb Douglas specification of the matching function

$$m(v, u, z) = g(z)Av^\alpha u^{1-\alpha}, \quad (12)$$

where $g'(\cdot) > 0$ and $g(0) = 1$. In turn, we have $\lambda_f = g(z)A(u/v)^{1-\alpha}$, and $\lambda_w = g(z)A(v/u)^\alpha$.

Since z does not enter (10), the productivity of the system is unaffected. In order to assess the effects of a change in z , on u and I , we first normalize (5 bis) dividing by $e^{\gamma t}$ in order to work with stationary variables.² Our system becomes

$$\begin{cases} \frac{u}{1-u} = \lambda_w^{-1}(\sigma + p(\vartheta, \lambda_f)) \\ \lambda_f V = c \end{cases}, \quad (13)$$

where $p(\vartheta, \lambda_f) \equiv \mu(\vartheta)/\mu$, and $c > 0$. Contrary to MW who assume $C'(I(t)) > 0$, and in line with the 'replication argument' according to which constant returns to scale should be assumed when all factors of production are variable, we assume $C'(I(t)) = 0$, and $C(I(t)) = ce^{\gamma t}$.

²Along a balanced growth path both $V(t)$ and $C(t)$ grow at the exogenous rate of technical change γ .

Solving the system (see Appendix) we obtain the equilibrium values for investment and unemployment rate:

$$I^* = \frac{1 + \frac{V}{c}(\sigma + p(\vartheta, c/V))}{\vartheta(1 + (\frac{c}{V})^{\frac{\alpha}{1-\alpha}}(\sigma + p(\vartheta, c/V))\left(\frac{1}{g(z)A}\right)^{\frac{1}{1-\alpha}})}, \quad (14)$$

$$u^* = \frac{(\frac{c}{V})^{\frac{\alpha}{1-\alpha}}(\sigma + p(\vartheta, c/V))\left(\frac{1}{g(z)A}\right)^{\frac{1}{1-\alpha}}}{\left(1 + (\frac{c}{V})^{\frac{\alpha}{1-\alpha}}(\sigma + p(\vartheta, c/V))\left(\frac{1}{g(z)A}\right)^{\frac{1}{1-\alpha}}\right)}. \quad (15)$$

We are now in the position to state the following:

Proposition 1 *an increase in z raises steady state equilibrium level of investment and lowers steady state equilibrium unemployment rate.*

Proof. See Appendix. ■

The positive effect on employment occurs as the improved matching efficiency raises the expected profitability and the equilibrium level of investment. Moreover, since the effect of the investment expansion is larger than the unemployment reduction, the policy brings about an increase in the number of vacancies i.e. $dv/dz = \vartheta(dI^*/dz) + du^*/dz > 0$ (see Appendix). Intuitively, decreasing returns to ζ in the probability of workers' matching λ_w prevents the increase in the level of investment to fully translate into employment. As a result of a higher v and a lower u , matching policies also reduce the average duration of unemployment spells, $1/\lambda_w$.

5 Concluding remarks

This paper reflects back on the relation between labor market institutions and technical change moving from the idea that centralized bargaining might favor job relocation to high tech activities. In particular, by partially isolating wages from plants productivity, centralized bargaining increases the relative profitability of more productive with respect to less productive ones. On the other hand, centralized bargaining fastens the scrapping of old plants and hence brings about structural unemployment. The negative impact of centralized bargaining on employment can be mitigated by wage moderation

or by policies favoring job relocation and retraining. To investigate the latter issue, we introduce labor market frictions in the benchmark Moene and Wallerstein model, and we find that relocation policies effectively improve investment and employment. Overall, our analysis suggests that policy interventions in systems characterized by centralized bargaining are essential to ensure a good performance in terms of employment.

The effectiveness of the labour market policies, however, depends on its design and the way it is financed, two issues that are left to future extensions of this paper. Concerning the former, retraining and active labor market policies tend to be less effective in periods of fast technological change because a larger technological distance among successive vintages accelerates the obsolescence of the specific skills acquired through retraining. As a result, in periods of fast technological progress, policies favouring job relocation should be designed so as to enhance the accumulation of general skills that are more adaptable to innovations (Krueger and Kumar 2004). The issue of how active labour market policies are financed turns out to be relevant when the policy is financed through a tax out of profits that would alter the investment expected profitability. In this case, the distribution of employment across firms of different age can be deformed by the policy itself, hence substantially increasing the complexity of our analysis.

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A Appendix I

Let us start with the system:

$$\begin{cases} \frac{u}{1-u} = \lambda_w^{-1}(\sigma + p(\vartheta, \lambda_f)) \\ \lambda_f V = c \end{cases}$$

From the second equation we have $\lambda_f = c/V$, so that we can re-write the system as

$$\begin{cases} \frac{u}{1-u} = \frac{1}{g(z)A} (u/v)^\alpha (\sigma + p(\vartheta), c/V) \\ g(z)A(u/v)^{1-\alpha}V = c \end{cases}.$$

From the second equation we have $u/v = (c/(g(z)AV))^{\frac{1}{1-\alpha}} = \delta (1/g(z)A)^{\frac{1}{1-\alpha}}$, where $\delta \equiv (c/V)^{\frac{1}{1-\alpha}}$, hence $u/1-u = \delta^\alpha \left(\frac{1}{g(z)A}\right)^{\frac{1}{1-\alpha}} (\sigma + p(\vartheta))$. Using the definition $v = \vartheta I - (1-u)$ in u/v to solve for u , we get $u = \delta (1/(g(z)A))^{\frac{1}{1-\alpha}} \cdot (\vartheta I - (1-u))$, whence $u = \delta (1/(g(z)A))^{\frac{1}{1-\alpha}} (\vartheta I - 1) / \left(1 - \delta (1/(g(z)A))^{\frac{1}{1-\alpha}}\right)$, $1-u = \left(1 - \delta \vartheta I \left(\frac{1}{g(z)A}\right)^{\frac{1}{1-\alpha}}\right) / \left(1 - \delta \left(\frac{1}{g(z)A}\right)^{\frac{1}{1-\alpha}}\right)$ and $u/1-u = \delta \left(\frac{1}{g(z)A}\right)^{\frac{1}{1-\alpha}} (\vartheta I - 1) / \left(1 - \delta \vartheta I \left(\frac{1}{g(z)A}\right)^{\frac{1}{1-\alpha}}\right)$. Equating the two expressions for $u/(1-u)$ solves for the equilibrium value of investment: $\delta(\vartheta I^* - 1) / \left(1 - \delta \vartheta I^* \left(\frac{1}{g(z)A}\right)^{\frac{1}{1-\alpha}}\right) = \delta^\alpha (\sigma + p(\vartheta))$.

Hence,

$$I^* = \frac{1 + \delta^{\alpha-1}(\sigma + p(\vartheta))}{\vartheta \left[1 + \delta^\alpha(\sigma + p(\vartheta)) \left(\frac{1}{g(z)A}\right)^{\frac{1}{1-\alpha}}\right]}.$$

In order to assess the effect of a change in labor market policies expenses on investment we differentiate investment with respect to z :

$$\frac{\partial}{\partial z} \left(\frac{1 + \delta^{\alpha-1}(\sigma + p(\vartheta))}{\vartheta \left[1 + \delta^\alpha(\sigma + p(\vartheta)) \left(\frac{1}{g(z)A}\right)^{\frac{1}{1-\alpha}}\right]} \right) = \frac{\delta^\alpha (p(\vartheta) + \sigma) \cdot g'_z \left(\frac{1}{Ag(z)}\right)^{\frac{\alpha}{1-\alpha}} [(p(\vartheta) + \sigma) \delta^{\alpha-1} + 1]}{\vartheta (1-\alpha) \cdot Ag(z)^2 \left[1 + \delta^\alpha(\sigma + p(\vartheta)) \left(\frac{1}{g(z)A}\right)^{\frac{1}{1-\alpha}}\right]^2} > 0.$$

Plugging I^* into $u = \delta \left(\frac{1}{g(z)A}\right)^{\frac{1}{1-\alpha}} (\vartheta I - 1) / \left(1 - \delta \left(\frac{1}{g(z)A}\right)^{\frac{1}{1-\alpha}}\right)$, we obtain the equilibrium value of unemployment $u^* =$

$$u^* = \frac{\delta \left(\frac{1}{g(z)A}\right)^{\frac{1}{1-\alpha}} \left[\frac{(1 + \delta^{\alpha-1}(\sigma + p(\vartheta))) \left(\frac{1}{g(z)A}\right)^{\frac{1}{1-\alpha}}}{\vartheta (1 + \delta^\alpha(\sigma + p(\vartheta)) \left(\frac{1}{g(z)A}\right)^{\frac{1}{1-\alpha}}) - 1} - 1 \right]}{1 - \delta \left(\frac{1}{g(z)A}\right)^{\frac{1}{1-\alpha}}} = \frac{\delta \left(\frac{1}{g(z)A}\right)^{\frac{1}{1-\alpha}} \left[\delta^{\alpha-1}(\sigma + p(\vartheta)) - \delta^\alpha(\sigma + p(\vartheta)) \left(\frac{1}{g(z)A}\right)^{\frac{1}{1-\alpha}} \right]}{\left[1 + \delta^\alpha(\sigma + p(\vartheta)) \left(\frac{1}{g(z)A}\right)^{\frac{1}{1-\alpha}}\right] \left(1 - \delta \left(\frac{1}{g(z)A}\right)^{\frac{1}{1-\alpha}}\right)} = \frac{\delta^\alpha(\sigma + p(\vartheta)) \left(\frac{1}{g(z)A}\right)^{\frac{1}{1-\alpha}}}{(1 + \delta^\alpha(\sigma + p(\vartheta)) \left(\frac{1}{g(z)A}\right)^{\frac{1}{1-\alpha}})}.$$

Differentiating unemployment with respect to z :

$$\frac{\partial}{\partial z} \left(\frac{\delta^\alpha (\sigma + p(\vartheta)) \left(\frac{1}{g(z)A} \right)^{\frac{1}{1-\alpha}}}{(1 + \delta^\alpha (\sigma + p(\vartheta)) \left(\frac{1}{g(z)A} \right)^{\frac{1}{1-\alpha}})} \right) = - \frac{\delta^\alpha (\sigma + p(\vartheta)) g'_z \cdot \left(\frac{1}{g(z)A} \right)^{\frac{\alpha}{1-\alpha}}}{(1 - \alpha) \cdot Ag(z)^2 \left[1 + \delta^\alpha (\sigma + p(\vartheta)) \left(\frac{1}{g(z)A} \right)^{\frac{1}{1-\alpha}} \right]^2} < 0.$$

Substituting $\delta \equiv (c/V)^{\frac{1}{1-\alpha}}$ in the expression for I^* , and u^* gives the values in the text.

Finally, the change in the number of vacancies is:

$$\partial v / \partial z = \vartheta (\partial I^* / \partial z) + \partial u^* / \partial z = \frac{\delta^{2\alpha-1} (p(\vartheta) + \sigma)^2 \cdot g'_z \left(\frac{1}{Ag(z)} \right)^{\frac{\alpha}{1-\alpha}}}{(1-\alpha) \cdot Ag(z)^2 \left[1 + \delta^\alpha (\sigma + p(\vartheta)) \left(\frac{1}{g(z)A} \right)^{\frac{1}{1-\alpha}} \right]^2} > 0.$$